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Linear Topologies on a Field and Completions of Valuation Rings

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Introduction.

For an integral local ring A, we consider the linear topology on QA with fundamental system of neighborhoods of 0:

$$\Sigma_A = \{a\mathfrak{m}(A) \mid a \in A, a \neq 0\}.$$

This topology is said to be the A-topology on QA. Here QA is the quotient field of A and $\mathfrak{m}(A)$ is the unique maximal ideal of A. In general, the A-topology is stronger than the $\mathfrak{m}(A)$-adic topology.

For an integral local ring A, we consider the completion

$$\hat{A} = \text{proj.lim}\, A/\mathfrak{a} \quad (\mathfrak{a} \in \Sigma_A)$$

with respect to the A-topology.

In this paper we shall study the fundamental properties of the completion $\hat{A}$ of an integral local ring A with respect to the A-topology and show some related examples. The A-topology and the completion $\hat{A}$ are very important conceptions for a valuation ring A, especially in the case that A is not noetherian. The main results are as follows:

THEOREM 1. *Let A be an integral local ring. Then*

$$A \text{ is a valuation ring} \Leftrightarrow \hat{A} \text{ is a valuation ring}.$$

Moreover, if A is a valuation ring, then the residue field of $\hat{A}$ is isomorphic to the residue field of A and the value group of $\hat{A}$ is isomorphic to the value group of A.

For a field K and a subring A of K, let $Zar(K|A)$ denote the set of valuation rings of K which contain A. Then the set $Zar(K|A)$ has a structure of local ringed spaces (see [4, §1]).

THEOREM 2. *Suppose that A is a valuation ring.*

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