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On the Volume Elements on an Expansive Set

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In [6], J. Moser proved that the group $\mathscr{D}(M)$ of all C^∞-diffeomorphisms of a compact connected C^∞-manifold M with $\partial M=\emptyset$ acts transitively on the space $\mathscr{V}$ of all C^∞-volume elements with total volume one, where the action is of course given by the pullback φ^*dV for $\varphi\in\mathscr{D}(M)$ and $dV\in\mathscr{V}$.

Moreover the mapping $\Phi\colon\mathscr{D}(M)\to\mathscr{V}$ given by $\Phi(\varphi)=\varphi^*dV$ for any fixed $dV\in\mathscr{V}$ defines a structure of principal fibre bundle with the fibre $\mathscr{D}_{dV}(M)=\{\varphi\in\mathscr{D}(M);\varphi^*dV=dV\}$, where the topologies are given by the C^∞-topology. Since $\mathscr{V}$ is convex, the above principal bundle turns out to be trivial, and hence $\mathscr{D}(M)$ is homeomorphic to $\mathscr{D}_{dV}(M)\times\mathscr{V}$ (cf. [8], [1], [9]). Especially, $\mathscr{D}_{dV}(M)$ is homotopically equivalent with $\mathscr{D}(M)$.

The purpose of this note is to show that a little weaker theorem holds for a wider class of compact sets, i.e., orientable expansive sets with nonvoid connected interior $'S$ such that $S=\overline{'S}$. Namely, in such a compact set S, the inclusion $i\colon\mathscr{D}_{dV}(S)\to\mathscr{D}(S)$ gives a *weak* homotopy equivalence.

§ 1. Preliminaries and the precise statement of the theorem.

Let N be an n-dimesional smooth (C^∞-) manifold and S a compact subset of N. By T'_S we denote the restriction of the tangent bundle T_N onto S. Functions, vector fields (sections of T'_S) or p-forms (sections of the exterior product $\bigwedge^p T'_S$) are said to be *smooth* if they can be extended smoothly on a neighborhood of S in N. A smooth vector field u on S is called a *strictly tangent vector field on* S if the integral curves of an extension $\tilde{u}$ of u with initial points in S are contained in S for $-\infty<t<\infty$. This property for u does not depend on the choice of extension $\tilde{u}$. By $\Gamma(T_S)$, we denote the totality of smooth strictly tangent vector fields on S. As it will be proved in the next section, $\Gamma(T_S)$ is a Lie algebra under the usual Lie bracket product and a $\Gamma(1_S)$-module, where $\Gamma(1_S)$ is the ring of all C^∞-functions on S.

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