TOKYO J. MATH.
VOL. 7, NO. 1, 1984

Asymptotic Behavior of Nonexpansive Mappings and Some Geometric Properties in Banach Spaces

Tomoyuki FUJIHIRA

Keio University

(Communicated by T. Saito)

Introduction

Throughout this paper, X denotes a real Banach space with the dual space X^* and the bidual space X^{**} and C is a closed convex subset of X. For $0\leqq\gamma\leqq 1$ we consider a mapping $T: C\to C$ such that $\|Tx-Ty\|\leqq \gamma\|x-y\|$ for all $x, y\in C$. A mapping T is called nonexpansive (resp. contraction) if $\gamma=1$ (resp. $\gamma<1$). Let $A\subset X\times X$ be an accretive operator satisfying the range condition $R(I+\lambda A)\supset \overline{D(A)}$ (the closure of the domain of A) for all $\lambda>0$, where I is the identity, $J_\lambda=(I+\lambda A)^{-1}$ be the resolvent, and let $A_\lambda=(I-J_\lambda)/\lambda$ be the Yosida approximation. A one-parameter family $\{T(t); t\geqq 0\}$ denotes the nonexpansive semigroup on $\overline{D(A)}$ generated by $-A$, i.e., $T(t)x=\lim_{\lambda\to 0+} J_\lambda^{[t/\lambda]}x$ for $x\in\overline{D(A)}$ and $t\geqq 0$ (see [1]). We use lim and w-lim for convergence in the strong and weak topology, respectively. We define $S(X)=\{x\in X; \|x\|=1\}$ and $d(0, R(A))=\inf\{\|x\|; x\in R(A)\}$, where $R(A)$ denotes the range of A.

Our main purpose is to show the following results.

THEOREM 1. *Let the sequence $\{x_n\}_{n\geqq 0}$ be defined by $x_{n+1}=c_nTx_n+(1-c_n)x_n$, where $x_0\in C$ and $\{c_n\}_{n\geqq 0}$ is a real sequence such that $0<c_n\leqq 1$ and $a_n=\sum_{i=0}^n c_i\to\infty$ as $n\to\infty$. Then there exists an $f\in S(X^*)$ such that for any $x, x_0\in C$,*

$$(1)\qquad \lim_{n\to\infty} f(T^nx)/n=\lim_{n\to\infty}\|T^nx\|/n=\inf_{y\in C}\|Ty-y\|$$
$$=\lim_{n\to\infty} f(x_{n+1})/a_n=\lim_{n\to\infty}\|x_{n+1}\|/a_n\ .$$

COROLLARY 2. (i) *In Theorem 1, if X is reflexive and strictly convex, then* w-$\lim_{n\to\infty} T^nx/n=$ w-$\lim_{n\to\infty} x_{n+1}/a_n=-v$ *for all $x, x_0\in C$, where $\|v\|=\inf_{y\in C}\|Ty-y\|$.*

(ii) *In Theorem 1, if X^* has Fréchet differentiable norm, then*

Received February 15, 1983