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A Diffusion Process with a One-Sided Brownian Potential

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Introduction.

Let $\mathbf{W}$ be the space of continuous functions w defined in $\mathbf{R}$ and vanishing identically on $[0, \infty)$. We denote by P the Wiener measure on $\mathbf{W}$, namely, P is the probability measure on $\mathbf{W}$ such that $\{w(-x), x \geq 0, P\}$ is a Brownian motion with time parameter x. Let $\Omega = \mathbf{C}([0, \infty); \mathbf{R})$ and write $X(t) = X(t, \omega) = \omega(t)$, where $\omega(t)$ is the value of $\omega(\in \Omega)$ at time t. Given $w \in \mathbf{W}$ and $x_0 \in \mathbf{R}$ we denote by $P_w^{x_0}$ the probability measure on Ω such that $\{X(t), t \geq 0, P_w^{x_0}\}$ is a diffusion process with generator

$$\mathcal{L}_w = \frac{1}{2} e^{w(x)} \frac{d}{dx} \left(e^{-w(x)} \frac{d}{dx} \right)$$

starting from x_0. Let $\mathcal{P}^{x_0}$ be the probability measure on $\mathbf{W} \times \Omega$ defined by

$$\mathcal{P}^{x_0}(dwd\omega) = P(dw) P_w^{x_0}(d\omega) .$$

The process $\{X(t), t \geq 0, \mathcal{P}^{x_0}\}$ is regarded as defined on the probability space $(\mathbf{W} \times \Omega, \mathcal{P}^{x_0})$, which we call a diffusion process with a one-sided Brownian potential. We are interested in the limiting behavior of $\{X(t), t \geq 0, \mathcal{P}^0\}$ as $t \to \infty$.

Our present model is a variant of the Brox-Schumacher diffusion ([1], [9]) that was introduced as a diffusion analogue of Sinai's random walk ([10]). When $w(x)$ does not vanish identically for $x \geq 0$, or more precisely speaking, when $\{w(x), x \geq 0, P\}$ and $\{w(-x), x \geq 0, P\}$ are independent Brownian motions, Brox [1] and Schumacher [9] proved that $\{(\log t)^{-2} X(t), t \geq 0, \mathcal{P}^0\}$ has a nondegenerate limit distribution. This result was extended to the case of a considerably wider class of (asymptotically) self-similar random environments by Kawazu, Tamura and Tanaka ([6], [7]). See [12] for a survey of results concerning diffusion processes in random environments. In our present model the random environment is self-similar but does not belong to the class of random environments of [6] because

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