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Remarks on Cusp Forms for Fricke Groups

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1. Introduction.

We consider the Teichmüller spaces of the closed torus and the once punctured torus. This is a part of a series of papers in which we investigate explicit relations between these spaces and we will give some remarks on our obtained results. In [A1] and [A2] we gave correspondences of subsets of these Teichmüller spaces and constructed holomorphic mappings between once punctured tori and closed tori based on these correspondences. In this paper we will show that constructions of these holomorphic mappings are closely related to constructions of cusp forms of weight 1.

First we recall a coordinate system for the Teichmüller space of the closed torus. We describe a closed torus by $R_\tau = \mathbf{C}/\Gamma_\tau$, $\Gamma_\tau = \{m + n\tau \mid m, n \in \mathbf{Z}\}$, then the Teichmüller space $\mathcal{T}_{1,0}$ of the closed torus is the upper half-plane $\mathbf{H}$, *i.e.,* a point in the Teichmüller space $\mathcal{T}_{1,0}$ of the closed torus is denoted by $\tau \in \mathbf{H}$. (See, for example, [IT].) We introduce the three subsets of $\mathcal{T}_{1,0}$: $L_1 = \{\tau \in \mathbf{H} \mid |\tau| \geq 1 \text{ and } \mathrm{Re}(\tau) = 0\}$, $L_2 = \{\tau \in \mathbf{H} \mid |\tau| = 1 \text{ and } -1/2 \leq \mathrm{Re}(\tau) \leq 0\}$ and $L_3 = \{\tau \in \mathbf{H} \mid |\tau| \geq 1 \text{ and } \mathrm{Re}(\tau) = -1/2\}$. These sets are characterized by the fact that in a fundamental domain for the modular group, $\tau \in L_1 \cup L_2 \cup L_3$ if and only if τ is a closed torus associated with a real lattice, that is, $\overline{\mu\Gamma_\tau} = \{\overline{\mu\gamma} \mid \mu\gamma \in \mu\Gamma_\tau\} = \mu\Gamma_\tau$ for some $\mu \in \mathbf{C}$.

Next we recall a coordinate system for the Teichmüller space of the once punctured torus. We use the convention that an element in PSL(2, $\mathbf{R}$) represents the Möbius transformation induced by it. In this paper we consider a Fuchsian group G consisting of Möbius transformations of PSL(2, $\mathbf{R}$) and having the following properties: (i) G is discontinuous in the upper half-plane $\mathbf{H}$, (ii) every real number is a limit point for G, (iii) G is finitely generated. A Fuchsian group $\Gamma = \langle A, B\rangle$ which is a free group generated by $A, B \in$ PSL(2, $\mathbf{R}$) is called a *Fricke group* if $X^2 + Y^2 + Z^2 = XYZ$ and $X, Y, Z > 2$, where $X = \mathrm{tr}\, A$, $Y = \mathrm{tr}\, B$ and $Z = \mathrm{tr}\, AB$. We consider a once punctured torus which is uniformized by a Fricke group Γ and take a normalized form for the representation of Γ (see [Sc]), then the Teichmüller space $\mathcal{T}_{1,1}$ of the once punctured torus can be identified with the set of all Fricke groups (see

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