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Central Extensions and Hasse Norm Principle over Function Fields

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0. Introduction.

Let K/k be a finite extension of global fields. Let $J(K)$ be the idele group of K and $N_{K/k}$ the norm map from K to k. We say that Hasse norm principle holds for K/k if $k^* \cap N_{K/k}J(K) = N_{K/k}K^*$.

In number field case, several authors have studied the validity of Hasse norm principle for abelian extensions. It is very closely tied up with central extensions. In [Ge2], Gerth gave necessary and sufficient conditions for Hasse norm principle to hold for cyclotomic fields. In [K], Kagawa gave conditions for Hasse norm principle to hold for maximal real subfields of cyclotomic fields. Central extensions are also useful in studying ideal class groups ([CoRo], [Fr], [Fu3]).

Let $k = \mathbf{F}_q(T)$ be the rational function field over finite field $\mathbf{F}_q$, where $q = p^f$, $p = \mathrm{char}(k)$ and $A = \mathbf{F}_q[T]$. For any monic polynomial $\mathfrak{m} \in A$, let $k(\Lambda_\mathfrak{m})$ be the $\mathfrak{m}$-th cyclotomic function field and $k(\Lambda_\mathfrak{m})^+$ its maximal real subfield.

In this paper, we define central class fields of Galois extensions of function fields, give necessary and sufficient conditions for Hasse norm principle to hold for $k(\Lambda_\mathfrak{m})$ and $k(\Lambda_\mathfrak{m})^+$, and find lower bounds for the ℓ-rank of ideal class groups of $k(\Lambda_\mathfrak{m})$ and $k(\Lambda_\mathfrak{m})^+$.

1. Central class field and Genus field.

Let k be a global function field over a finite field $\mathbf{F}_q$. Let ∞ be a place of degree 1 of k and $\mathcal{O}_k$ the ring of regular elements outside ∞ of k. Let E_k be the unit group of $\mathcal{O}_k$, which is just $\mathbf{F}_q^*$. We write k_∞ to be the completion of k at ∞. We fix a sing function $sgn : k_\infty^* \to \mathbf{F}_q^*$ and choose a uniformizer π of k_∞ with $sgn(\pi) = 1$. Denote by $\tilde{C}$ the field $k_\infty(\sqrt[q-1]{-\pi})$. In the following we mean by an extension of k, a separable extension of k for which any embeddings into k_∞^{ac} lies in $\tilde{C}$ viewing as a subfield of k_∞^{ac}.

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