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Distinguished Bases of Non-simple Singularities

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Dedicated to Professor Takuo Fukuda on his sixtieth birthday

0. Introduction.

Let $f : (\mathbf{C}^n, \mathbf{0}) \to (\mathbf{C}, \mathbf{0})$ be an arbitrary function germ, with an isolated critical point at zero. Let $\Delta_1, \Delta_2, \cdots, \Delta_\mu$ be a distinguished basis of vanishing cycles in the homology group $H_{n-1}(V_\varepsilon; \mathbf{Z}) \cong \mathbf{Z}^\mu$ of the non-singular level manifold. With respect to such a basis the variation operator Var (resp. Var^{-1}) of the singularity f is represented by an upper triangular matrix. In [5], Gusein-Zade gave the following converse result for simple singularities.

GUSEIN-ZADE THEOREM 1. *Let $f : (\mathbf{C}^n, \mathbf{0}) \to (\mathbf{C}, \mathbf{0})$ be one of the simple singularities A_k, D_k, E_6, E_7 and E_8 and $\Delta_1, \Delta_2, \cdots, \Delta_\mu$ be an integral basis in the homology group $H_{n-1}(V_\varepsilon; \mathbf{Z}) \cong \mathbf{Z}^\mu$, in which the matrix of the operator Var (resp. Var^{-1}) is upper triangular. Then $\Delta_1, \Delta_2, \cdots, \Delta_\mu$ is a distinguished basis of vanishing cycles.*

For the proof of this, the following result for simple singularities is used which is of interest in its own right.

GUSEIN-ZADE THEOREM 2. *Let $f : (\mathbf{C}^n, \mathbf{0}) \to (\mathbf{C}, \mathbf{0})$ be one of the simple singularities A_k, D_k, E_6, E_7 and E_8 in an odd number of variables n. For any vanishing cycle Δ and any distinguished basis $\Delta_1, \Delta_2, \cdots, \Delta_\mu$ for f, there exists a sequence of elementary substitutions, turning it into a distinguished basis $\Delta'_1, \Delta'_2, \cdots, \Delta'_\mu$ with the first element $\Delta'_1 = \pm\Delta$.*

In [3] page 103, V. I. Arnol'd et als propose as an open problem to study whether an analogous theorem to Gusein-Zade Theorem 2 is true for non-simple singularities. The purpose of the present paper is to give a negative answer to this problem. Two distinguished bases of vanishing cycles in the homology group $H_{n-1}(V_\varepsilon; \mathbf{Z})$ are said to be *elementary equivalent* if one of the two bases can be transfered into the other by a (finite) sequence of elementary substitutions and changing of the orientation of some of the elements of the basis. Then the main theorem in this paper can be stated as follows.

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