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On a Fluctuation Identity for Multidimensional Lévy Processes

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1. Introduction.

Fluctuation identities for one-dimensional Lévy processes, often called Wiener Hopf factorizations, were investigated by many authors (e.g. Baxter and Donsker [1], Rogozin [9], Gusak and Korolyuk [5], Pecherskii and Rogozin [8], Borovkov [3], Greenwood and Pitman [4], Skorohod [11, §**4.3** and **4.4**], Sato [10, Chapter 9], Bertoin [2, Chapter VI]). There are several types of identities, for instance, those concerning supremum processes mainly and including ladder processes too. We are particularly interested in those given by Pecherskii and Rogozin [8], involving supremum processes, and developed in Sato [10, Chapter 9]. Our aim is to give some extension of their results to multidimensional Lévy processes. Our problem might be discussed from the view-point of Millar's general results ([6] [7]) on the decomposition of Markov processes at splitting times but detailed computations would be needed to arrive at our result. In this paper, we employ an elementary method starting from random walks; it may be a straightforward extension of the method developed in Sato[10, pp. 333–345] but we emphasize that there is a crucial point concerning a careful definition of $X''(H^-(t)*)$ and $X''(H^+(t)*)$ (see §**2** and Lemma 5.4). Another emphasis is that a relevant choice of an approximating compound Poisson process much simplifies the argument of deriving the result for general Lévy processes from that for compound Poisson processes (see (5.2) and Lemma 5.4; compare it with the arguments of [11, pp. 207–213] and [10, pp. 342–345]). The method of approximation is, in the sense of analysis, the same as the well-known method, often called Yosida's approximation, in the theory of semigroups of linear operators which makes use of a bounded operator $\varepsilon^{-1}(\varepsilon^{-1}I - A)^{-1}A$ to approximate an unbounded infinitesimal generator A ([12]).

2. The main result.

Given a Lévy process $X(t)$ taking values on $\mathbf{R}^d$, $d \geq 2$, with $X(0) = 0$, we denote by $X'(t)$ and $X''(t)$ the first and the other components of $X(t)$, respectively, and so $X(t)$ can be

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