On the rank of the elliptic curve $y^2 = x^3 + kx$

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In this note, we consider the elliptic curve

$$\varepsilon_k : y^2 = x^3 + kx.$$

Mestre showed in [2] that there are infinitely many values of $k \in \boldsymbol{Q}$, for which the rank of ε_k is at least 4. Nagao showed the same result in [3] by a different construction. We shall improve this result in this paper.

(See Theorem 2 below.)

Let

$$k(t) = -16(-2+t^2)^2(2+2t+t^2)^2 (6+4t+t^2)(2+4t+3t^2)* (16+56t+86t^2+68t^3+31t^4+8t^5+t^6)* (8+32t+62t^2+68t^3+43t^4+14t^5+2t^6)* (28+136t+296t^2+368t^3+287t^4+146t^5 +49t^6+10t^7+t^8)* (32+176t+460t^2+680t^3+612t^4+340t^5 +115t^6+22t^7+2t^8)* (48+224t+492t^2+656t^3+572t^4+328t^5 +123t^6+28t^7+3t^8)* (64+320t+784t^2+1168t^3+1148t^4+ 736t^5+296t^6+68t^7+7t^8).$$

We consider the following elliptic curve

$$\varepsilon_{k(t)} : y^2 = x^3 + k(t)x$$

$\varepsilon_{k(t)}$ have 5 $\boldsymbol{Q}(t)$-rational points $P_i = (x_i, y_i)$ $(1 \leq i \leq 5)$, where

$$x_1 = -4(-2+t^2)^2(2+2t+t^2)^4(6+4t+t^2) (2+4t+3t^2)* (16+56t+86t^2+68t^3+31t^4+8t^5+t^6)* (8+32t+62t^2+68t^3+43t^4+14t^5+2t^6),$$

$$y_1 = 8(-2+t^2)^2(2+2t+t^2)^3(6+4t+t^2)(2 +4t+3t^2)* (16+56t+86t^2+68t^3+31t^4+8t^5+t^6)* (8+32t+62t^2+68t^3+43t^4+14t^5+2t^6)* (1536+15872t+78656t^2+246720t^3+ 546512t^4+904288t^5+1153680t^6+ 1155360t^7+916600t^8+577680t^9+288420t^{10} +113036t^{11}+34157t^{12}+7710t^{13}+1229t^{14} +124t^{15}+6t^{16}),$$

$$x_2 = -4(-2+t^2)(6+4t+t^2)(2+4t+3t^2)* (16+56t+86t^2+68t^3+31t^4+8t^5+t^6)* (28+136t+296t^2+368t^3+287t^4+146t^5 +49t^6+10t^7+t^8)* (32+176t+460t^2+680t^3+612t^4+340t^5 +115t^6+22t^7+2t^8),$$

$$y_2 = 8(-2+t^2)^2(6+4t+t^2)(2+4t+3t^2)* (16+56t+86t^2+68t^3+31t^4+8t^5+t^6)* (28+136t+296t^2+368t^3+287t^4+146t^5 +49t^6+10t^7+t^8)* (32+176t+460t^2+680t^3+612t^4+340t^5 +115t^6+22t^7+2t^8)* (192+1696t+6840t^2+16704t^3+27476t^4 +32080t^5+27318t^6+17168t^7+7947t^8+ 2658t^9+613t^{10}+88t^{11}+6t^{12}),$$

$$x_3 = (-2+t^2)(6+4t+t^2)(2+4t+3t^2)* (8+32t+62t^2+68t^3+43t^4+14t^5+2t^6)* (32+176t+460t^2+680t^3+612t^4+340t^5 +115t^6+22t^7+2t^8)* (64+320t+784t^2+1168t^3+1148t^4+ 736t^5+296t^6+68t^7+7t^8),$$

$$y_3 = (-2+t^2)^2(6+4t+t^2)(2+4t+3t^2)* (8+32t+62t^2+68t^3+43t^4+14t^5+2t^6)* (32+176t+460t^2+680t^3+612t^4+340t^5 +115t^6+22t^7+2t^8)* (64+320t+784t^2+1168t^3+1148t^4+ 736t^5+296t^6+68t^7+7t^8)* (768+5632t+19616t^2+42528t^3+63576t^4 +68672t^5+54636t^6+32080t^7+13738t^8+ 4176t^9+855t^{10}+106t^{11}+6t^{12}),$$

$$x_4 = -4(-2+t^2)^2(2+2t+t^2)(2+4t+3t^2)* (48+224t+492t^2+656t^3+572t^4+328t^5 +123t^6+28t^7+3t^8)* (64+320t+784t^2+1168t^3+1148t^4+ 736t^5+296t^6+68t^7+7t^8),$$

$$y_4 = 8(-2+t^2)^2(2+2t+t^2)^2(2+4t+3t^2)* (48+224t+492t^2+656t^3+572t^4+328t^5 +123t^6+28t^7+3t^8)* (64+320t+784t^2+1168t^3+1148t^4+ 736t^5+296t^6+68t^7+7t^8)* (576+5120t+21616t^2+56912t^3+ 103600t^4+137144t^5+135656t^6+101764t^7 +58308t^8+25506t^9+8431t^{10}+2054t^{11}+ 351t^{12}+38t^{13}+2t^{14}),$$

$$x_5 = -4t^4(-2+t^2)^2(2+2t+t^2)(6+4t+$$