A Three-manifold Invariant Derived from the Universal Vassiliev-Kontsevich Invariant†)

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We construct a three manifold invariant from the universal Vassiliev-Kontsevich (UVK) invariant $\hat{Z}$, which includes Lescop's generalization of the Casson-Walker invariant.

The UVK invariant $\hat{Z}$ has values in the space $\mathscr{A}$ of chord diagrams subject to the four term relation [2, 1, 3]. Here, we construct a three manifold invariant by taking Kirby move (Fig. 1) invariant part from the UVK invariant of framed links $\hat{Z}_f$ constructed in [3]. We modify $\hat{Z}_f$ so that it has a good property with respect to the KII moves. Let $\nu = Z_f(U)^{-1}$, which is the factor introduced in [3] to normalize the effect of maximal and minimal points. For an l-component link L, let $\check{Z}_f(L) = \hat{Z}_f(L) \# (\nu, \nu, \cdots, \nu)$. This means that we connect-sum ν to each string of $\hat{Z}_f(L)$. Then, we take a certain quotient $\bar{\mathscr{A}}$ of $\mathscr{A}$ so that the image of $\check{Z}_f(L)$ is stable under the KII moves. Let $\Lambda'(L)$ denote this image of $\check{Z}_f(L)$, then $\Lambda'(L)$ factors the Jones-Witten invariant in [6, 9] except the normalization factor for the KI moves. We study a low degree part of $\Lambda'(L)$ concretely, and, after normalizing for the KI moves, we show that $\Lambda'(L)$ includes the order of the first homology and Casson's invariant. For a $\mathbf{Z}/r\mathbf{Z}$-homology 3-sphere (r: odd prime), the Jones-Witten invariant dominates the Casson-Walker invariant [7].

In this short note, we expose the results and the idea of proofs. The detail will be given elsewhere.

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1. Modified universal Vassiliev-Kontsevich invariant. We use notations in [3, 4]. Let C be a chord diagram with a distinguished string s, and let k be the number of end point of chords on s. Let $\Delta(C)$ denote the sum of 2^k diagrams obtained by adding a string parallel to s and changing each point on s as in Fig. 2.

Proposition 1. *Let L and L' be two links as in the KII move in Fig.* 1. *Then $\check{Z}_f(L')$ is obtained from $\check{Z}_f(L)$ as in Fig.* 3.

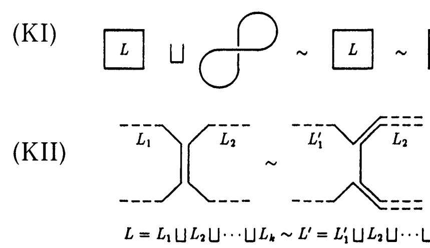

Fig. 1. Kirby Moves

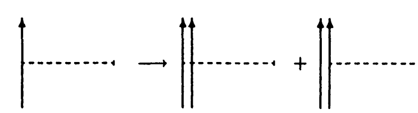

Fig. 2. Parallel of a chord diagram.

To prove this proposition, we use the result in [4].

Let $\mathscr{A}^{(l)}$ denote the $\mathbf{C}$-linear space spanned by the chord diagrams on a disjoint union of l S^1's subject to the four term relation. We add two types of relations to $\mathscr{A}^{(l)}$. The first one is for orientations of strings. Let D be a chord diagram and let D' be a chord diagram obtained by changing the orientation of a string s of D. Then we impose $D' \sim (-1)^{e(s)} D$, where $e(s)$ denote the number of end points on s. We call this the orientation independence relation. The second one is for the KII move given in Fig. 4. We call it the KII relation of chord diagrams. Let $\bar{\mathscr{A}}^{(l)} = \mathscr{A}^{(l)}/$(Orientation independence relation, KII relation), and let $\Lambda'(L)$ be the image of $\check{Z}_f(L)$ in $\bar{\mathscr{A}}^{(l)}$