3. *K-theoretic Groups with Positioning Map and Equivariant Surgery**),**)

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1. Introduction. Since 1990 the authors have studied equivariant surgery on manifolds allowing half dimensional singular sets. The full detail of obtained theory [3] is rather complicated. The purpose of this article is to present an outline of the theory for use in Transformation Groups. We treat material here in a restrictive way comparing with [3] in order to make the paper easy reading. However we will describe the theory so far as one can have important geometric applications. The following two theorems are examples of such applications.

Theorem 1.1 ([3]). *A standard sphere S has a smooth, one fixed point action of some finite group if and only if the dimension of S is greater than* 5. *Moreover, if* $\dim S > 5$ *then S has such an exotic action of A_5* (*the alternating group of degree* 5).

Background of this theorem is explained in [2], [4], and [10]. The corresponding assertion in the category of locally linear actions was proven in [4].

After [11], we denote by $\mathcal{G}_p^q$ the class of all finite groups G having series of normal subgroups $P \lhd H \lhd G$ such that $|P|$ is a power of p, H/P is cyclic, and $|G/H|$ is a power of q.

Theorem 1.2 ([8]). *A finite group G admits a smooth, one fixed point action on a standard sphere of some dimension if and only if $G \notin \mathcal{G}_p^q$ for any primes p and q.*

This theorem was proven by [12] under the hypothesis that G is abelian of odd order. It was also shown in [9] that any finite nonsolvable group G admits such exotic actions.

In the current paper G will be a finite group, R will be $\boldsymbol{Z}$ (the ring of integers), $\boldsymbol{Z}_{(p)}$ (the localization of $\boldsymbol{Z}$ at a prime p) or $\boldsymbol{Q}$ (the ring of rational numbers), and $A = R[G]$ will be the group ring of G with coefficients in R.

2. Grothendieck-Witt rings. Let Θ be a finite G-set. A G-map α from Θ to a finitely generated A-module M is called a *Θ-positioning map* of M. If Θ consists of a unique point then (M, α) is nothing but a pointed module.

In order to generalize the ordinary Grothendieck-Witt ring $GW(R, G)$ (cf. [1] or [6]), we introduce the category $\boldsymbol{H}_{G-inv}(R, \Theta)$ as follows. The

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