71. *Convolution Semigroups of Stable Distributions over a Nilpotent Lie Group*

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We study stable properties of convolution semigroups of probability distributions over a Lie group. Stable distributions over a Heisenberg group or more generally on a homogeneous group were studied by Hulanicki [3], Glowacki [1] and others. Our stable distribution is motivated by these works. However, our definition is more general than their's, thereby including all strictly operator-stable distributions in case where the underlying group is a Euclidean space.

1. Convolution semigroup of probability distributions. Let G be a Lie group of dimension d. Elements of G are denoted by σ, τ etc. Let $\mathcal{G}$ be its left invariant Lie algebra, where an inner product $\langle\ ,\ \rangle$ and the associated norm $|\ |$ are defined, so that it can be identified with an Euclidean space $\boldsymbol{R}^d$. Elements of $\mathcal{G}$ are denoted by X, Y etc. We fix its basis $\{X_1, \dots, X_d\}$. Let C be the set of all continuous maps from the Lie group G into $\boldsymbol{R} = (-\infty, \infty)$ (such that $\lim_{\sigma\to\infty} f(\sigma)$ exists if G is non compact, where ∞ is the infinity). It is a Banach space by the supremum norm. We denote by C^2 the totality of $f \in C$ such that it is twice continuously differentiable and Xf, YZf belong to C for any X, Y, Z.

Let μ be a probability distribution over G. Let $\varphi : G \to G$ (or $G \to \mathcal{G}$ or $\mathcal{G} \to G$) be a continuous map. The transformation of μ by φ is defined by $\varphi\mu(A) = \mu(\varphi^{-1}(A))$. For two distributions μ and ν, their convolution is a distribution on G defined by $\mu * \nu(A) = \int_G \mu(d\sigma)\nu(\sigma^{-1}A)$. The n-ple convolution of the distribution μ is denoted by μ^{n*}.

A family of probability distributions $\{\mu_t\}_{t>0}$ over the Lie group G is called a *convolution semigroup* (*of probability distributions*), if it satisfies (i) $\mu_s * \mu_t = \mu_{s+t}$ for all s, $t > 0$, and (ii) μ_h converges weakly to δ_e as $h \to 0$, where δ_e is the unit measure at the unit element e of G.

Suppose that we are given a convolution semigroup of probability distributions $\{\mu_t\}_{t>0}$ over G. We set for $f \in C$, $T_t f(\tau) = \int_G f(\tau\sigma)\mu_t(d\sigma)$. Then $\{T_t\}_{t>0}$ defines a semigroup of strongly continuous linear operators on the Banach space C. The infinitesimal generator L of $\{T_t\}_{t>0}$ is often called the *infinitesimal generator of* $\{\mu_t\}_{t>0}$. Hunt [4] has shown that the domain of the infinitesimal generator L includes C^2 and represented Lf, $f \in C^2$ by making use of the basis of the Lie algebra $\mathcal{G}$ and a Lévy measure on the Lie group G. We shall obtain another representation of the infinitesimal gener-