35. *An Example of Elliptic Curve over Q(T) with Rank $\geq$ 13*

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Abstract: We construct an elliptic curve over $Q(T)$ with rank ≥ 13.

In [1](resp. [2]), Mestre constructed elliptic curves over $Q(T)$ with rank ≥ 11 (resp. 12). In this paper, we construct an elliptic curve over $Q(T)$ with rank ≥ 13 using Mestre's method. As was explained in [1], for any 6-ple

$A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) \in Z^6$, we put $q_A(X) = \Pi_{i=1}^{6}(X - \alpha_i)$ and put $p_A(X) = q_A(X - T) * q_A(X + T) \in Q(T)[X]$. Then there are $g_A(X)$ and $r_A(X) \in Q(T)[X]$ with $\deg g_A = 6$, $\deg r_A \leq 5$ such that $p_A = g_A^2 - r_A$. Then the curve $Y^2 = r_A(X)$ contains 12 $Q(T)$-rational points $P_1, \ldots, P_{12}$ where $P_i = (T + \alpha_i, g_A(T + \alpha_i))$, $P_{i+6} = (-T + \alpha_i, g_A(-T + \alpha_i))$ $(1 \leq i \leq 6)$.

Let c_5 be the coefficient of X^5 of $r_A(X)$. By a suitable choice of A, we can assume that $c_5 = 0$. In the following, A will be always chosen so that $c_5 = 0$. Then $Y^2 = r_A(X)$ gives an elliptic curve over $Q(T)$ which will be denoted by $\mathscr{E}_A$.

Now, let $A = (148,116,104,57,25,0)$. (Then we have $c_5 = 0$.) In this case, the equation of the curve $\mathscr{E}_A$ and its $Q(T)$-rational points $P_1, \ldots, P_{12}$ are written as follows.

$$
\begin{aligned}
Y^2 = {} & (9T^2 + 211950)X^4 + (-2700T^2 - 63901710)X^3 + \\
& (-18T^4 + 396150T^2 + 6706476489)X^2 + (2700T^4 - \\
& 29575350T^2 - 284435346600)X + 9T^6 - 159200T^4 + \\
& 891699592T^2 + 4156297690000,
\end{aligned}
$$

$$
\begin{aligned}
P_1 &= [T + 148,\ 662T^2 + 66873T + 1868944] \\
P_2 &= [T + 116,\ -554T^2 - 39687T - 191632] \\
P_3 &= [T + 104,\ -526T^2 - 28497T + 163372] \\
P_4 &= [T + 57,\ 508T^2 - 19332T - 368809] \\
P_5 &= [T + 25,\ 580T^2 - 49116T + 566825] \\
P_6 &= [T,\ -670T^2 + 69759T - 2038700] \\
P_7 &= [-T + 148,\ -662T^2 + 66873T - 1868944] \\
P_8 &= [-T + 116,\ 554T^2 - 39687T + 191632] \\
P_9 &= [-T + 104,\ 526T^2 - 28497T - 163372] \\
P_{10} &= [-T + 57,\ -508T^2 - 19332T + 368809] \\
P_{11} &= [-T + 25,\ -580T^2 - 49116T - 566825] \\
P_{12} &= [-T,\ 670T^2 + 69759T + 2038700].
\end{aligned}
$$

By a direct calculation, we see that $\mathscr{E}_A$ contains another $Q(T)$-rational point

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