18. *Periodic Solutions of the 2-dimensional Heat Convection Equations*

By Kazuo ŌEDA

Faculty of Science, Japan Women's University

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§1. Introduction. We consider the heat convection equation in a time-dependent domain $\Omega(t) \subset R^2$. We assume that the domain $\Omega(t)$ varies periodically in t with period T. In the 3-dimensional case, we proved the existence and uniqueness of the periodic strong solution in [7] and showed the stability of it in [8] when the data were small in a suitable sense. In this paper, under somewhat released conditions than 3-dimensional case, we have studied the existence, uniqueness and the stability of the periodic strong solution. Recently, Morimoto [5] has got the periodic weak solution and Inoue-Ôtani [3] obtained the periodic strong one under their various situations.

§2. Assumptions and formulation. Let $\Omega(t)$ be a time-dependent bounded space domain in R^2 with the boundary $\partial\Omega(t) = \Gamma_0 \cup \Gamma(t)$, where Γ_0 is the inner boundary and $\Gamma(t)$ is the outer one. We denote by K the compact set which is bounded by Γ_0. We suppose that $\Omega(t)$ is included in a fixed open ball B_1 with radius d such that $\Omega(t) \subset B_1$. We make the following assumptions:

(A0) Γ_0 and $\Gamma(t)$ do not intersect each other.

(A1) For each fixed $t > 0$, $\Gamma(t)$ and Γ_0 are both simple closed curves and they are of class C^3.

(A2) $\Gamma(t) \times \{t\}\,(0 < t < T)$ changes smoothly (say, of class C^4) with respect to t.

(A3) $g(x)$ is a bounded and continuous vector function in $R^2 \setminus \operatorname{int} K$.

(A4) $\beta(x, t)$ is defined on $\partial\Omega(t)$ and it can be extended to a vector function $b = b(x, t)$ of the form $b = \operatorname{rot} c$, where $c(x, t)$ is defined in $B \times [0, \infty)$, of class C^3 and periodic in t with period T. Moreover, it satisfies the following condition

$$\int_{\Gamma_i} \beta \cdot n \, ds = 0, \; i = 0, 1,$$

where Γ_1 means $\Gamma(t)$ and n is the outer normal vector to $\partial\Omega(t)$.

(A5) The domain $\Omega(t)$, the boundary $\Gamma(t)$ and the function $\beta(x, t)$ vary periodically in t with period $T > 0$. i.e., $\Omega(t + T) = \Omega(t)$, $\Gamma(t + T) = \Gamma(t)$ and $\beta(\cdot, t + T) = \beta(\cdot, t)$ for each $t > 0$.

Now, let $u = u(x, t)$, $\theta = \theta(x, t)$ and $p = p(x, t)$ be the velocity of the viscous fluid, the temperature and pressure, respectively. Furthermore, let $\nu, \kappa, \alpha, \rho$ be physical constants and $g = g(x)$ be the gravitational vector. Then we consider the heat convection equation (HC) of Boussinesq