17. *New Criteria for Multivalent Meromorphic Starlike Functions of Order Alpha*

By M. K. Aouf

Department of Mathematics, Faculty of Science, University of Mansoura, Egypt

(Communicated by Kiyosi Itô, M. J. A., March 12, 1993)

Abstract: Let $M_{n+p-1}(\alpha)$ ($p \in N = \{1, 2, \ldots\}$, $n > -p$. $0 \leq \alpha < p$) deonte the class of functions of the form

$$f(z) = \frac{1}{z^p} + \frac{a_0}{z^{p-1}} + \frac{a_1}{z^{p-2}} + \cdots$$

which are regular and p-valent in the punctured disc $U^* = \{z : 0 < |z| < 1\}$ and satisfy the condition

$$\mathrm{Re}\left\{\frac{D^{n+p}f(z)}{D^{n+p-1}f(z)} - (p+1)\right\} < -\frac{p(n+p-1)+\alpha}{n+p}, \ |z| < 1,$$

$0 \leq \alpha < p$, where

$$D^{n+p-1}f(z) = \frac{1}{z^p(1-z)^{n+p}} * f(z) \quad (n > -p).$$

It is proved that $M_{n+p}(\alpha) \subset M_{n+p-1}(\alpha)$ $(0 \leq \alpha < p,\ n > -p)$. Since $M_0(\alpha)$ is the class of p-valent meromorphically starlike functions of order $\alpha(0 \leq \alpha < p)$, all functions in $M_{n+p-1}(\alpha)$ are p-valent meromorphically starlike functions of order α. Further we consider the integrals of functions in $M_{n+p-1}(\alpha)$.

1. Introduction. Let Σ_p denote the class of functions of the form

$$f(z) = \frac{1}{z^p} + \frac{a_0}{z^{p-1}} + \frac{a_1}{z^{p-2}} + \ldots \quad (p \in N = \{1,2\ldots\}) \tag{1.1}$$

which are regular and p-valent in the punctured disc $U^* = \{z : 0 < |z| < 1\}$ and let n be any integer greater than $-p$. A function $f(z)$ in Σ_p is said to be p-valent meromorphically starlike of order $\alpha(0 \leq \alpha < p)$ if and only if

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} < -\alpha \quad \text{for } |z| < 1. \tag{1.2}$$

The Hadamard product or convolution of two functions f, g in Σ_p will be denoted by $f * g$. Let

$$D^{n+p-1}f(z) = \frac{1}{z^p(1-z)^{n+p}} * f(z) \quad (n > -p) \tag{1.3}$$

$$= \frac{1}{z^p}\left[\frac{z^{n+2p-1}f(z)}{(n+p-1)!}\right]^{(n+p-1)} \tag{1.4}$$

$$= \frac{1}{z^p} + \frac{n+p}{z^{p-1}}a_0 + \frac{(n+p)(n+p+1)}{2!\,z^{p-2}}a_1 + \cdots. \tag{1.5}$$

AMS (1980) Subject Classification. 30C45-30C50.