16. *Some Generalizations of the Unicity Theorem of Nevanlinna*

By Nobushige TODA

Department of Mathematics, Nagoya Institute of Technology

(Communicated by Kiyosi ITÔ, M. J. A., March 12, 1993)

1. Introduction. Let $f(z)$ be a transcendental meromorphic function in $|z| < \infty$ and let $S(f)$ be the set of meromorphic functions $a(z)$ in $|z| < \infty$ which satisfy

$$T(r, a) = o(T(r, f)) \quad (r \to \infty).$$

We consider $\bar{C} = C \cup \{\infty\}$ to be a subset of $S(f)$. We put for $a \in S(f)$

$$E(f = a) = \{z : f(z) - a(z) = 0\}.$$

More than sixty years ago, R. Nevanlinna proved the following theorem, which is called the Unicity Theorem.

Theorem A. *Let f_1 and f_2 be transcendental meromorphic functions in $|z| < \infty$. If for five distinct values $a_1, \ldots, a_5$ of $\bar{C}$*

$$E(f_1 = a_j) = E\{f_2 = a_j) \quad (j = 1, \ldots, 5),$$

then $f_1 = f_2$ ([2], p. 109, see also [1], p. 48).

The following theorem was used to prove Theorem A in [2].

Theorem B. *For any $q(\geq 3)$ distinct values $a_1, \ldots, a_q$ of $\bar{C}$,*

$$(1) \qquad (q-2)T(r, f) < \sum_{j=1}^{q} \bar{N}(r, a_j) + S(r, f)$$

([2], p. 70).

The functions $f_1(z) = e^z$, $f_2(z) = e^{-z}$, with $a_1 = 0$, $a_2 = 1$, $a_3 = -1$ and $a_4 = \infty$ show that Theorem A is best ([2], p. 111).

It is an open problem to generalize Theorem A to the case when $a_1, \ldots, a_5$ belong to $S(f)$ ([3]). This is neither trivial nor easy since we do not have an inequality corresponding to (1) for $a_1, \ldots, a_q$ of $S(f)$ except when $q = 3$. When $q = 3$, we have the following theorem.

Theorem C. *Suppose that a_1, a_2 and a_3 are distinct in $S(f)$. Then we have*

$$(1 + o(1))T(r, f) < \sum_{j=1}^{3} \bar{N}(r, 1/(f - a_j)) + S(r, f)$$

as $r \to \infty$ (see [1], p. 47).

It is a very interesting open problem whether (1) holds for distinct $a_1, \ldots, a_q$ in $S(f)$ ([1], p. 47; cf. [4], Satz 1).

The purpose of this paper is to give some generalizations of Theorem A by making use of Theorem C. We use the standard notation of the Navanlinna theory of meromorphic functions ([1], [2]) and we use ${}_nC_k = n!/(n-k)!k!$ as the binomial coefficient.

2. Lemmas. We shall give some lemmas in this section. Let f be a transcendental meromorphic function in $|z| < \infty$.