# 14. A Generalization of Tate-Nakayama Theorem by Using Hypercohomology

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**§1. Introduction and notations.** In the classical class field theory, the isomorphism theorem is proved by using Tate's cohomology and Tate-Nakayama theorem [1], and similar methods are used to prove the isomorphism theorem of higher dimensional class field theory (cf. [2], [3] and [4]). Especially, the proof of the isomorphism theorem of class field theory of two dimensional local fields, as given in [2], looks like the classical one by using generalized Tate-Nakayama theorem and modified hypercohomology, which is a very satisfactory generalization of Tate's cohomology.

For higher dimensional class field theory, further generalization of Tate-Nakayama theorem seems to be of great interest. This was partially achieved in [2], where this theorem was proved for two-term complexes. The aim of this paper is to prove it for arbitrary bounded complexes.

Unless the contrary is explicitly stated, we shall employ the following notation and convention throughout this paper: all groups are finite and all complexes are bounded. Let $G$ be a group and $M$ a $G$-module. We denote $M^G$ by $\Gamma(G, M)$, which is viewed as a functor. We shall freely use the standard notations on complexes and objects in derived categories as in [2], [3] and [4]. For example, for a complex $A^{\cdot}$ and an integer $m$, we define a new complex $A^{\cdot}[m]$ by $(A[m])^q = A^{q+m}$.

**§2. The generalized Tate-Nakayama theorem.** As a preparation, we recall the definition and basic properties of modified hypercohomology.

Consider an exact sequence

$$\cdots \to X^{-2} \to X^{-1} \to X^0 \to X^1 \to X^2 \to \cdots$$

such that

(1) Each term $X^n$ is a free $\boldsymbol{Z}[G]$-module with a finite basis.

(2) The sequence

$$\cdots \to X^{-2} \to X^{-1} \to X^0 \to \boldsymbol{Z} \to 0$$

is a projective resolution of the $G$-module $\boldsymbol{Z}$ with trivial action.

Such an exact sequence is called a *complete resolution* of $G$.

It is a well-known fact that for any $G$-module $M$, the cohomology groups of the complex:

$$\cdots \to \mathrm{Hom}_G(X^1, M) \to \mathrm{Hom}_G(X^0, M) \to \mathrm{Hom}_G(X^{-1}, M) \to \cdots$$

coincides with Tate's cohomology groups.

Note that in the definitions of usual hypercohomology, we consider the double complex $\bigoplus_{i \geq 0} Y^{i,j}$ such that

$$Y^{i,j} = \mathrm{Hom}_G(X^{-j}, A^j),$$