94. *On a Relative Normal Integral Basis Problem over Abelian Number Fields**)

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We say that a Galois extension L/K of a number field K with Galois group G has a *relative normal integral basis* (RNIB, for short) when the integer ring O_L of L is free over the group ring $O_K[G]$. Let p be a prime number and assume that K contains a primitive p-th root of unity. In [3], Childs proved that an unramified cyclic extension L/K of degree p has an RNIB if and only if L is obtained by adjoining to K a p-th root of a unit of K satisfying a certain congruence. Let $\mathcal{H}(K)$ be the subgroup of $K^{\times}/K^{\times p}$ consisting of classes $[\alpha]$ ($\alpha \in K^{\times}$) for which $K(\alpha^{1/p})$ is unramified over K, and $\mathcal{N}(K)$ be the subgroup of $\mathcal{H}(K)$ consisting of classes $[\alpha]$ ($\in \mathcal{H}(K)$) for which the unramified cyclic extension $K(\alpha^{1/p})/K$ has an RNIB. Using the above result and tools of Iwasawa theory, we shall describe, in terms of power series attached to p-adic L-functions, the Galois module structure of the quotient $\mathcal{H}(K)/\mathcal{N}(K)$ when the base field K runs over all layers of the cyclotomic $\boldsymbol{Z}_p$-extension of a certain imaginary abelian field (Theorem). As a corollary, we give a necessary and sufficient condition for $\mathcal{H}(K) = \mathcal{N}(K)$ for such K in terms of an Iwasawa invariant and a certain distinguished polynomial. Though there are several results to the effect that relative Galois extensions have no RNIB (e.g. Fröhlich[7, Chap. 6, §3], Cougnard[4], Brinkhuis[1]), there seems to be few results in the other derection. An immediate consequence of the Corollary is that any unramified cyclic extension of degree p over K as above has an RNIB if the base field K is a "sufficiently" high layer. This paper is an announcement of the results generalizing those of our paper [10]. The details will appear elsewhere.

Let p be a fixed odd prime number and k be an imaginary abelian field satisfying the following three conditions.

(C1) k contains a primitive p-th root of unity.

(C2) $p \nmid [k:\boldsymbol{Q}]$.

(C3) There is only one prime ideal of k over p.

Typical examples of such k are (1) $k = \boldsymbol{Q}(\mu_p)$, and (2) $p = 3$, $k = \boldsymbol{Q}(\sqrt{-3}, \sqrt{d})$ where d is a rational integer with $d \equiv 2 \pmod{3}$. Let k_∞/k be the cyclotomic $\boldsymbol{Z}_p$-extension and k_n be its n-th layer ($n \geq 0$). Put $\Delta = \mathrm{Gal}(k/\boldsymbol{Q})$ and $\Gamma = \mathrm{Gal}(k_\infty/k)$. We write, for brevity, $\mathcal{H}_n = \mathcal{H}(k_n)$ and $\mathcal{N}_n = \mathcal{N}(k_n)$. The Galois groups Δ and Γ act on these groups naturally. Let Ψ be an irreducible character of Δ over $\boldsymbol{Q}_p$. We call such Ψ a $\boldsymbol{Q}_p$-character. We fix an irreducible component ψ of Ψ over an algebraic closure Ω_p of $\boldsymbol{Q}_p$, which we

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