55. *A Remark on the Limiting Absorption Method for Dirac Operators**)

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1. **Introduction and result.** Let us consider the Dirac operator

$$H = \sum_{j=1}^{3} \alpha_j D_j + \beta + q(x),\ x \in \boldsymbol{R}^3,\ D_j = -i\frac{\partial}{\partial x_j},$$

in the Hilbert space $[L^2(\boldsymbol{R}^3)]^4$, where α_j and $\alpha_4 = \beta$ are 4×4 Hermitian constant matrices satisfying the anti-commutation property

$$\alpha_j \alpha_k + \alpha_k \alpha_j = 2\delta_{jk} I \quad (1 \le j,\ k \le 4),$$

and $q(x)$ is a continuous real valued function which decays at infinity, where I is the unit 4×4 matrix. For a real number t, let $L^2_t(\boldsymbol{R}^N)$ be the weighted Hilbert space with the norm

$$\| f \|_t = \left\{ \int_{\boldsymbol{R}^N} (1 + |x|^2)^t \, |f(x)|^2 \, dx \right\}^{1/2} < \infty$$

and let X_t be a the weighted Hilbert space defined by $[L^2_t(\boldsymbol{R}^3)]^4$ (,where we use also the same notation $\|\ \ \|_t$ as the norm). One can see by the limiting absorption method under appropriate conditions on $q(x)$ that

for any $t > \frac{1}{2}$ *and any* $f \in X_t$ *the strong limit of the resolvent*

$$R(\lambda \pm i\,0) f = \text{s} - \lim_{\varepsilon \to +0} (H - \lambda \mp i\varepsilon)^{-1} f \quad \textit{in } X_{-t}$$

exists for any real λ *such that* $|\lambda| > 1$ (see, e. g., Yamada [6]).

For Schrödinger operators $h = -\Delta + q(x)$ in $\boldsymbol{R}^N$ there are also many works on the limiting absorption method, which shows that

for any $t > \frac{1}{2}$ *and any* $f \in L^2_t(\boldsymbol{R}^N)$ *the strong limit of the resolvent*

$$r(\lambda \pm i\,0) f = \text{s-}\lim_{\varepsilon \to +0} (h - \lambda \mp i\varepsilon)^{-1} f \quad \textit{in } L^2_{-t}(\boldsymbol{R}^N)$$

exists for any $\lambda > 0$.

Let us dentote the operator norm of $r(\lambda \pm i\,0)$ ($R(\lambda \pm i\,0)$) as a bounded operator on L^2_t to L^2_{-t} (on X_t to X_{-t}) by $\| r(\lambda \pm i\,0) \|_{t,-t}$ ($\| R(\lambda \pm i\,0) \|_{t,-t}$).

It is well known that the operator norm $\| r(\lambda \pm i\,0) \|_{t,-t}$ for each $t > \frac{1}{2}$ satisfies

$$\| r(\lambda \pm i\,0) \|_{t,-t} = O(\lambda^{-1/2}) \text{ as } \lambda \to \infty$$

for Schrödinger operators with a large class of potentials (see, e.g., Saitō [3],

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