52. *Large-time Existence of Surface Waves of Compressible Viscous Fluid*

By Naoto TANAKA*) and Atusi TANI**)

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1. Introduction and theorem. In this communication we are concerned with free boundary problem for compressible viscous isotropic Newtonian fluid which is formulated as follows: Find the domain $\Omega_t \subset \boldsymbol{R}^3$ occupied by the fluid at the moment $t > 0$ together with the density $\rho(x, t)$, velocity vector field $v(x, t) = (v_1, v_2, v_3)$ and the absolute temperature $\theta(x, t)$ satisfying the system of Navier-Stokes equations

$$(1.1) \quad \begin{cases} \dfrac{D\rho}{Dt} + \rho(\nabla\cdot v) = 0, \quad \rho\dfrac{Dv}{Dt} = \nabla\cdot\boldsymbol{P} - \rho g e_3, \\ \rho c_V \dfrac{D\theta}{Dt} + \theta p_\theta(\nabla\cdot v) = \nabla\cdot(\kappa\nabla\theta) + \Psi \\ (x \in \Omega_t \equiv \{x' = (x_1, x_2) \in \boldsymbol{R}^2, -b(x') < x_3 < F(x', t)\},\ t > 0) \end{cases}$$

and the initial and boundary conditions

$$(1.2) \quad \begin{cases} (\rho, v, \theta)|_{t=0} = (\rho_0, v_0, \theta_0) \quad (x \in \Omega_0), \\ \boldsymbol{P}n = -p_e n + \sigma H n, \quad \kappa\nabla\theta\cdot n = \kappa_e(\theta_e - \theta) \\ (x \in \Gamma_t \equiv \{x' \in \boldsymbol{R}^2, x_3 = F(x', t)\},\ t > 0), \\ v = 0, \ \theta = \theta_a \quad (x \in \Sigma \equiv \{x' \in \boldsymbol{R}^2, x_3 = -b(x')\},\ t > 0), \\ \dfrac{D}{Dt}(x_3 - F) = 0 \quad (x \in \Gamma_t, t > 0), \quad F|_{t=0} = F_0(x') \quad (x' \in \boldsymbol{R}^2). \end{cases}$$

Here $\nabla = \left(\dfrac{\partial}{\partial x_1}, \dfrac{\partial}{\partial x_2}, \dfrac{\partial}{\partial x_3}\right)$; $\nabla' = (\nabla_1, \nabla_2) = \left(\dfrac{\partial}{\partial x_1}, \dfrac{\partial}{\partial x_2}\right)$; $\dfrac{D}{Dt} = \dfrac{\partial}{\partial t} + (v\cdot\nabla)$ is the material derivative; $\boldsymbol{P} = (-p + \mu'(\nabla\cdot v))\boldsymbol{I} + 2\mu\boldsymbol{D}(v) \equiv -p\boldsymbol{I} + \boldsymbol{V}$ is the stress tensor; $\boldsymbol{I}$ is the 3×3 unit matrix; $\boldsymbol{D}(v)$ is the veloctiy deformation tensor with the elements $D_{ij} = \dfrac{1}{2}\left(\dfrac{\partial v_i}{\partial x_j} + \dfrac{\partial v_j}{\partial x_i}\right)$; $\Psi = \mu'(\nabla\cdot v)^2 + 2\mu\boldsymbol{D}(v) : \boldsymbol{D}(v)$ is the dissipation function; $p = p(\rho, \theta)$ is the pressure with $p_\rho, p_\theta > 0$; $(\mu, \mu', \kappa, c_V)(\rho, \theta)$ are, respectively, coefficient of viscosity, second coefficient of viscosity, coefficient of heat conductivity, heat capacity at constant volume, which are all assumed to be known smooth functions of (ρ, θ) satisfying $\mu, \kappa, c_V > 0$, $2\mu + 3\mu' \geq 0$; $(g, \sigma, p_e, \kappa_e)$ are, respectively, acceleration of gravity, coefficient of surface tention, atmospheric pressure, coefficient of outer heat conductivity, which are all assumed to be positive constants; $e_3 = {}^t(0, 0, 1)$; $n = \dfrac{1}{\sqrt{1 + |\nabla' F|^2}}\ {}^t(-\nabla_1 F,$

*) Department of Mathematics, Waseda Universtity.
**) Department of Mathematics, Keio University.