51. *On the Structure of Painlevé Transcendents with a Large Parameter*[†)]

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§0. Introduction. The purpose of this note is to report a novel and intriguing result (Theorem 3.2 below) on the structure of Painlevé transcendents with a large parameter, which asserts that they can be locally reduced to a solution of the Painlevé I with a large parameter η (cf. Table 1.1 below). The details shall be published elsewhere.

The Painlevé equations with a large parameter to be discussed here naturally arise as conditions for isomonodromic deformations (in the sense of Jimbo and Miwa [3]) of certain Schrödinger equations (with a large parameter η) tabulated in §1. We hope the result reported here will turn out to be effective not only for the better understanding of the Painlevé transcendents but also for the computation of the monodromic structures of those equations in terms of WKB solutions (cf. [5], [2]). We sincerely thank Professors T. Aoki and M. Jimbo for the stimulating discussions on these topics, from which we have benefited much.

§1. List of Painlevé equations with a large parameter and associated Schrödinger equations. In order to fix our notations we list up Painlevé equations with a large parameter (Table 1.1) and the relevant Schrödinger equations (Table 1.2). The latter ones can be isomonodromically deformed if the unknown function $\psi_J (J=I,\dots,VI)$ satisfies the deformation equation

$$\frac{\partial \psi_J}{\partial t} = A_J(x,t,\lambda)\frac{\partial \psi_J}{\partial x} - \frac{1}{2}\frac{\partial A(x,t,\lambda)}{\partial x}\psi_J, \tag{1.1}$$

where A_J is the rational function tabulated in Okamoto [4], §4.4 (without the subscript J); for example,

$$A_J = \frac{1}{2(x-\lambda)} \quad (J=I,\, II), \tag{1.2}$$

$$A_{VI} = \frac{\lambda - t}{t(t-1)}\frac{x(x-1)}{x-\lambda}, \text{ etc.} \tag{1.3}$$

where λ is a solution of the Painlevé equation P_J tabulated below. Here and in what follows we use the symbol P_J to denote the J-th Painlevé equation with a large parameter η as specified below, although they differ from the original Painlevé equations in that they contain a large parameter. Similarly the symbol SL_J in Table 1.2 below denotes the Schrödinger equation with a large parameter. The parameter η is introduced into these equations in such

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