# 50. *Error Estimation of Newmark's Method for Conservative Second Order Linear Evolution Equation*

By Mihoko MATSUKI and Teruo USHIJIMA

Department of Computer Science and Information Mathematics,
The University of Electro-Communications

(Communicated by Kiyosi ITÔ, M. J. A., Sept. 13, 1993)

**1. Introduction.** We consider the following continuous problem in a Hibert space $X$ :

$$(E : \varphi^1, \varphi^0, \psi) \qquad \begin{cases} \dfrac{d^2}{dt^2}\varphi + A\varphi = \psi,\ t > 0, \\ \varphi(0) = \varphi^1,\ \dfrac{d\varphi}{dt}(0) = \varphi^0 \end{cases}$$

with a positive definite selfadjoint operator $A$, and its fully discrete approximate problem in a given finite dimensional subspace $X_h$ of $X$ obtained by Newmark's method as follows:

$(E_{h,\tau} : \varphi^1_{h,\tau}, \varphi^0_{h,\tau}, \psi_{h,\tau,m})$

$$\begin{cases} \dfrac{\varphi_{h,\tau,m-1} - 2\varphi_{h,\tau,m} + \varphi_{h,\tau,m+1}}{\tau^2} \\ \qquad + A_h\{(1-2\beta+\delta)\varphi_{h,\tau,m} + \beta\varphi_{h,\tau,m+1} + (\beta-\delta)\varphi_{h,\tau,m-1}\} \\ \qquad = \psi_{h,\tau,m}, \qquad m = 1,2,\cdots, \\ \varphi_{h,\tau,0} = \varphi^1_{h,\tau},\quad \dfrac{\varphi_{h,\tau,1} - \varphi_{h,\tau,0}}{\tau} = \varphi^0_{h,\tau} \end{cases}$$

with a bounded positive definite selfadjoint operator $A_h$, where $\beta$ and $\delta$ are fixed nonnegative numbers independent of $h \in (0, \bar{h}]$, and $\tau$ is a positive number. For the derivation of the problem $(E_{h,\tau})$, we followed Raviart and Thomas [4].

Our motivation of this study is to analyze the finite element approximation of the linear water wave equation:

$$(\text{LWW}) \qquad \begin{cases} -\Delta\Phi = 0 & \text{in } \Omega,\ t > 0, \\ \Phi_{tt} + g\dfrac{\partial\Phi}{\partial n} = F_t & \text{on } \Gamma_0,\ t > 0, \\ \dfrac{\partial\Phi}{\partial n} = 0 & \text{on } \Gamma_1,\ t > 0, \\ \Phi(0, x) = \Phi^1(x) & \text{on } \Gamma_0, \\ \Phi_t(0, x) = \Phi^0(x) & \text{on } \Gamma_0, \end{cases}$$

where $\partial\Omega = \overline{\Gamma_0 \cup \Gamma_1}$ with mutually disjoint portions $\Gamma_0$ and $\Gamma_1$ of the boundary $\partial\Omega$ of the water region $\Omega$ at rest. The portion $\Gamma_0$ is the water surface at rest, and $\Gamma_1$ is the rigid wall. The problem (LWW) describes the motion of the water in a vessel on the ground of the Earth under the assumption of infinitesimal amplitude, and whose derivation from the fundamental laws of