49. *Singular Variation of Non-linear Eigenvalues. II*

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Let M be a bounded domain in R^3 with smooth boundary ∂M. Let w be a fixed point in M. Removing an open ball $B(\varepsilon\,;w)$ of radius ε with the center w from M, we get $M_\varepsilon = M\setminus\overline{B(\varepsilon\,;w)}$. For $p>1$ and $\varepsilon>0$ let $\lambda(\varepsilon)$ denote the positive number defined by

$$(1.1)_\varepsilon \qquad \lambda(\varepsilon) = \inf_{X_\varepsilon}\int_{M_\varepsilon}|\nabla u|^2\,dx,$$

where $X_\varepsilon = \{u\in H_0^1(M_\varepsilon) : \|u\|_{L^{p+1}(M_\varepsilon)} = 1,\ u\geq 0\}$.
We consider the asymptotic behaviour of $\lambda(\varepsilon)$ as ε tends to 0. It is well known that there exists at least one positive solution u_ε which attains $(1.1)_\varepsilon$ in case of $p\in(1,5)$. We note that the minimizer satisfies $-\Delta u_\varepsilon = \lambda(\varepsilon)u_\varepsilon^p$ in M_ε and $u_\varepsilon = 0$ on ∂M_ε. We put

$$\lambda = \inf_X\int_M|\nabla u|^2\,dx,$$

where $X = \{u\in H_0^1(M) : \|u\|_{L^{p+1}(M)} = 1,\ u\geq 0\}$.

In this paper we show the following

Theorem 1. *Assume that the positive solution of* $-\Delta u = \lambda u^p$ *in* M *under the Dirichlet condition on* ∂M *is unique. Then, there exists a constant* $p^*(M)>1$ *such that for any* $p\in(1,p^*(M))$ *we have*

$$(1.2) \qquad \lambda(\varepsilon)-\lambda = 4\pi\varepsilon u(w)^2 + o(\varepsilon)$$

as ε *tends to zero.*

Example. $M = B(r)$, the ball of radius r, satisfies the assumption of Theorem 1, as is seen in Gidas-Ni-Nirenberg [1, Theorem 1 and p. 224, 2.9]. See also Dancer [2, Theorem 5].

Theorem 1 follows from the following Theorems 2 and 3.

Theorem 2 (Ozawa [5]). *Fix* $p\in(1,5)$. *Assume that the positive solution of* $-\Delta u = \lambda u^p$ *in* M *under the Dirichlet condition on* ∂M *is unique. Moreover assume that* $\mathrm{Ker}(A+\lambda p u^{p-1}) = \{0\}$, *where we denote* A *by the linear operator* $H^2(M)\cap H_0^1(M)\ni u\to\Delta u\in L^2(M)$. *Then,* (1.2) *holds.*

Theorem 3. *Assume that the positive solution of* $-\Delta u = \lambda u^p$ *in* M *under the Dirichlet condition on* ∂M *is unique. Then, there exists* $p^*(M)>1$ *such that* $\mathrm{Ker}(A+\lambda p u^{p-1}) = \{0\}$ *holds for* $p\in(1,p^*(M))$.

We consider the eigenvalue problem (1.3).

$$(1.3) \qquad \begin{aligned} -\Delta\varphi &= \mu u^{p-1}\varphi \quad \text{in } M\\ \varphi &= 0 \qquad\quad\ \ \text{in } \partial M. \end{aligned}$$

Let $\mu_1^{(p)}$ ($\mu_2^{(p)}$, respectively) be the first (the second, respectively) eigenvalue of (1.3). Let $\varphi_1^{(p)}$ be the first eigenfunction of (1.3) which is normalized as

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