3. Generalization of a Theorem of Manin-Shafarevich*)

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Let us fix some notation before stating the results. Let m be a positive integer with $m \geq 3$. Let $\{\Gamma_t \mid t \in \boldsymbol{P}^1\}$ be a linear pencil of curves of degree m in a projective plane $\boldsymbol{P}^2$ defined over an algebraically closed field k of arbitary characteristic. Assume the following conditions:

(A1) Every member Γ_t is irreducible and general members are nonsingular.

(A2) The m^2 base points of the pencil are distinct. We denote them by $P_i (i = 0,1,\ldots,m^2 - 1)$.

Then the generic member $\Gamma = \Gamma_t$ (for t generic over k) is a nonsingular curve of genus $g = (m-1)(m-2)/2$ defined over the rational function field $K = k(t)$.

Let J denote the Jacobian variety of Γ/K and $J(K)$ the group of its K-rational points. Each P_i defines a K-rational point of Γ. By choosing one of P_i, say P_0, we have a natural embedding of Γ into J sending P_0 to the origin of J. Thus we have

$$P_1, \ldots, P_{m^2-1} \in \Gamma(K) \subset J(K).$$

For $m = 3$, $\{\Gamma_t\}$ is a pencil of cubic curves and $J = \Gamma$ is an elliptic curve, say E, over K. Inspired by Shafarevich, Manin proved that under (A1) and (A2) the 8 points $P_1, \ldots, P_8$ are independent and generate a subgroup of index 3 in the Mordell-Weil group $E(K)$ (see [5], Th.6 and [6], Ch.IV, 26.4). Recently we have given a simple proof of this result based on the theory of Mordell-Weil lattices, where $E(K)$ is endowed with the structure of the root lattice E_8 (see [7], Th. 10.11).

More recently we have extended the notion of Mordell-Weil lattices to higher genus case [9]. As an application, we can prove the following result generalizing the above theorem of Manin-Shafarevich to arbitary $m \geq 3$.

Theorem 1. *The notation being as above, assume the conditions* (A1) *and* (A2). *Then the group of K-rational points $J(K)$ of the Jacobian variety J is a torsionfree abelian group of rank $r = m^2 - 1$, and the r points $P_i (1 \leq i \leq r)$ are independent and generate a subgroup of index m in $J(K)$.*

This is an immediate consequence of Theorem 2 below formulated in terms of Mordell-Weil lattices. By blowing up the m^2 base points from $\boldsymbol{P}^2$, we obtain a nonsingular rational surface S and a morphism

$$f : S \to \boldsymbol{P}^1$$

such that $f^{-1}(t) \simeq \Gamma_t (t \in \boldsymbol{P}^1)$. In particular, Γ/K is the generic fibre of this genus g fibration f. The exceptional curves (P_i) in S arising from

*) Dedicated to I. R. Shafarevich for his 70th birthday.