2. *Curves and Symmetric Spaces*

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This is an announcement of our research on the classification of curves, i.e., compact Riemann surfaces, of genus $g=7$, 8 and 9 and their canonical rings by means of the symmetric spaces $X_{2g-2}^{24-2g}\subset \boldsymbol{P}^{22-g}$ studied in [3]. The details will be published elsewhere. A line bundle L on a curve C is a g_d^r if $\deg L=d$ and $\dim H^0(C, L)\geq r+1$.

§ 1. Linear section theorems. A non-hyperelliptic curve C embedded in $\boldsymbol{P}^{g-1}$ by the canonical linear system $|K_C|$ is called a *canonical curve*. The canonical ring of C is isomorphic to the homogeneous coordinate ring of $C\subset \boldsymbol{P}^{g-1}$ by Noether's theorem.

Let $X_{12}^8\subset \boldsymbol{P}^{14}$ be the 8-dimensional complex Grassmannian $U(6)/(U(2)\times U(4))$ embedded in $\boldsymbol{P}^{14}$ by the Plücker coordinates. It is classically known that a transversal linear subspace P of dimension 6 cut out a curve C of genus 8 and that the embedding $C\subset P$ is canonical.

Theorem 1. *A curve C of genus 8 is a transversal linear section of the 8-dimensional Grassmannian if and only if C has no g_7^2.*

Complex Grassmannians are symmetric spaces of type AIII. Besides $X_{14}^8\subset \boldsymbol{P}^{14}$ two compact Hermitian symmetric spaces $X_{12}^{10}\subset \boldsymbol{P}^{15}$ and $X_{16}^6\subset \boldsymbol{P}^{13}$ yield canonical curves (of genus 7 and 9) as transversal linear sections. The former is $SO(10)/U(5)$ of type DIII embedded in the projectivization of the space U^{16} of semi-spinors. Let $\mathrm{Alt}_5\, \boldsymbol{C}$ be the space of skew-symmetric matrices of degree 5. Then $X_{12}^{10}\subset \boldsymbol{P}^{15}$ is the compactification of the embedding

$$\begin{array}{ccc} \mathrm{Alt}_5\, \boldsymbol{C} & \longrightarrow & \boldsymbol{P}^{15}, \\ \cup\!\!\!\!\!\shortmid & & \cup\!\!\!\!\!\shortmid \\ A=(a_{ij}) & \longmapsto & (1 : a_{12} : \cdots : a_{45} : \mathrm{Pfaff}\, A^1 : \cdots : \mathrm{Pfaff}\, A^5) \end{array}$$

where $A^1, \cdots, A^5$ are the principal minors of A. The latter is the compact dual $Sp(3)/U(3)$ of the Siegel upper half space $\mathfrak{H}_3$ of degree 3 embedded in the projectivization of a 14-dimensional irreducible representation U^{14} of $Sp(3)$. Let $\mathrm{Sym}_3\, \boldsymbol{C}$ be the space of symmetric matrices of degree 3. Then $X_{16}^6\subset \boldsymbol{P}^{13}$ is the compactification of the Veronese-like embedding

$$\begin{array}{ccc} \mathrm{Sym}_3\, \boldsymbol{C} & \longrightarrow & \boldsymbol{P}(\boldsymbol{C}\oplus \mathrm{Sym}_3\, \boldsymbol{C}\oplus \mathrm{Sym}_3\, \boldsymbol{C}\oplus \boldsymbol{C}), \\ \cup\!\!\!\!\!\shortmid & & \cup\!\!\!\!\!\shortmid \\ A & \longmapsto & (1 : A : A' : \det A) \end{array}$$

where A' is the cofactor matrix of A.

Theorem 2. *A curve C of genus* 7 (resp. 9) *is a transversal linear section of $X_{12}^{10}\subset \boldsymbol{P}^{15}$* (resp. $X_{16}^6\subset \boldsymbol{P}^{13}$) *if and only if C has no g_5^1* (resp. g_6^1).