70. *Examples of Elliptic Curves over Q with Rank ≥ 17*

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Abstract: We construct some elliptic curves over Q with *rank* ≥ 17.

Recently, Mestre [1,2,3] constructed large rank elliptic curves. We summarize his results in the following three propositions.

Proposition 1 ([1]). *Let $\mathfrak{E}$ be the curve over rational function field $Q(T)$ of the form*

$$Y^2 = (429T^2 + 55260)X^4 - (5434T^2 + 1239000)X^3 + (-3432T^4 - 2451T^2 + 1222156)X^2 + (21736T^4 - 3637984T^2 + 134780352)X + 6864T^6 - 1074992T^4 + 53200096T^2 - 758849264.$$

Then $Q(T)$-rank of $\mathfrak{E}$ is ≥ 11.

Proposition 2 ([2]). *Let $\mathfrak{E}'$ be the curve over $Q(T')$ ($Q(T')$ also means rational function field) which is obtained by specializing $\mathfrak{E}$ to $T = (3T'^2 - 478T' + 1287)/(T'^2 - 429)$. Then $Q(T')$-rank of $\mathfrak{E}'$ is ≥ 12.*

Proposition 3 ([3]). *Let C be the curve defined over Q which is obtained by specializing $\mathfrak{E}'$ to $T' = 77$. Then Q-rank of C is ≥ 15.*

Let N be a fixed positive integer. For an elliptic curve E defined over Q, we define ([4])

$$S = S(N) = \sum (2 + a_p)\log p/(p + 1 - a_p)$$
$$S' = S'(N) = \sum - a_p \log p$$

where $a_p = p + 1 - \# E(F_p)$ and p moves over prime numbers satisfying $p \leq N$. We experimentally know that elliptic curves whose S and S' are sufficiently large have large ranks ([4]).

For a rational number t, let E_t be an elliptic curve defined over Q which is obtained by specializing $\mathfrak{E}$ to $T = t$. We consider the family of elliptic curves $\{E_{t_1/t_2}\}$ where (t_1, t_2) moves over coprime integers satisfying $1 \leq t_1 \leq 1000$ and $1 \leq t_2 \leq 100$. By selecting the curves in this family satisfying $S_{401} > 39$, $S_{1009} > 54$, and, $S'_{1009} > 17000$, we obtain four curves $E_{967/59}$, $E_{866/35}$, $E_{542/49}$, and, $E_{537/71}$.

Theorem. (1) *Q-rank of $E_{537/71}$ is ≥ 17.*
(2) *Q-rank of $E_{866/35}$ is ≥ 17.*

Proof of (1). $E_{537/71}$ is Q-isomorphic to the following minimal Weiestrass curve of the form $y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$ where

$a_1 = 1$
$a_2 = 0$
$a_3 = 0$
$a_4 = -18957824833624761882478 25431$

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