8. Certain Differential Operators for Meromorphically p-valent Convex Functions

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Abstract: Let $J_n(\alpha)$ be the class of functions of the form

$$f(z)=\frac{a_{-p}}{z^p}+\sum_{k=0}^{\infty} a_k z^k \quad (a_{-p}\neq 0,\, p\in N=\{1,2,\cdots\})$$

which are regular in the punctured disk $E=\{z: 0<|z|<1\}$ and satisfying

$$\mathrm{Re}\left\{\frac{(D^{n+1}f(z))'}{(D^n f(z))'}-(p+1)\right\}<-p\frac{n+\alpha}{n+1} \quad (n\in N_0=\{0,1,2,\cdots\}, |z|<1, 0\leq\alpha<1),$$

where

$$D^n f(z)=\frac{a_{-p}}{z^p}+\sum_{m=1}^{\infty}(p+m)^n a_{m-1} z^{m-1}.$$

It is proved that $J_{n+1}(\alpha)\subset J_n(\alpha)$. Since $J_0(\alpha)$ is the class of meromorphically p-valent convex functions of order α, all functions in $J_n(\alpha)$ are p-valent convex. Futher properties preserving integrals are considered.

1. Introduction. Let $\sum_p$ denote the class of functions of the form

$$f(z)=\frac{a_{-p}}{z^p}+\sum_{k=0}^{\infty} a_k z^k \quad (a_{-p}\neq 0,\, p\in N=\{1,2,\cdots\}) \tag{1.1}$$

which are regular in the punctured disk $E=\{z: 0<|z|<1\}$. Define

$$D^0 f(z)=f(z), \tag{1.2}$$

$$D^1 f(z)=\frac{a_{-p}}{z^p}+(p+1)a_0+(p+2)a_1 z+(p+3)a_2 z^2+\cdots =\frac{(z^{p+1}f(z))'}{z^p}, \tag{1.3}$$

$$D^2 f(z)=D(D^1 f(z)), \tag{1.4}$$

and for $n=1,2,\cdots$,

$$D^n f(z)=D(D^{n-1}f(z))=\frac{a_{-p}}{z^p}+\sum_{m=1}^{\infty}(p+m)^n a_{m-1}z^{m-1} =\frac{(z^{p+1}D^{n-1}f(z))'}{z^p}. \tag{1.5}$$

In this paper, we shall show that a function $f(z)$ in $\sum_p$, which satisfies one of the conditions

$$\mathrm{Re}\left\{\frac{(D^{n+1}f(z))'}{(D^n f(z))'}-(p+1)\right\}<-p\frac{n+\alpha}{n+1}, \quad (z\in U=\{z: |z|<1\}), \tag{1.6}$$

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