62. *Existence of a Rational Elliptic Surface with a Given Mordell-Weil Lattice*

By Tetsuji SHIODA

Department of Mathematics, Rikkyo University

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In this note, we announce some results concerning the existence of a rational elliptic surface having a given structure of the Mordell-Weil lattice, which has been classified in [5]. With some arithmetic applications in mind, we consider the question over the rational number field $\boldsymbol{Q}$. Details will be given in forthcoming papers. For general facts on Mordell-Weil lattices (MWL), we refer to [7] or [8].

1. Notation. Let $K = k(t)$ be the rational function field over an algebraically closed ground field k, and let E/K be an elliptic curve such that the associated elliptic surface (the Kodaira-Néron model)

$$f : S \to \boldsymbol{P}^1$$

is a rational elliptic surface. Then the structure of the Mordell-Weil lattice $E(K)$ (by which we mean, by abuse of language, the structure of the Mordell-Weil group $E(K)$ equipped with the height pairing) is completely determined by the "trivial lattice" T formed by irreducible components of reducible singular fibres $f^{-1}(v)$ (let R be the set of such v's); T is the direct sum of simple root lattices T_v of type A, D, E and has a natural embedding into the root lattice E_8:

$$T = \oplus_{v \in R} T_v \hookrightarrow E_8. \tag{1}$$

Namely, if we denote by $L = E(K)^0$ the narrow Mordell-Weil lattice, then L is isomorphic to the orthogonal complement of T in E_8, while $M = E(K)$ is isomorphic to the direct sum of L^* (the dual lattice of L) and a finite torsion group (see [8], Th. 10.3 or [5], Th. 3.1; we follow the latter notation here).

Further the possible structure of the triple $\{T, L, M\}$ has been classified into 74 types: No.1,...,No.74 ([5], Main Theorem).

Remark. (i) The terminology "trivial lattice" was used in [7], [8] to mean the lattice generated by T as above, the zero section and any fibre in the Néron-Severi lattice of S. The present usage is more convenient for the purpose of this note, and we hope it will cause no confusion.

(ii) We take this opportunity to correct the misprints in the table of [5]: For No.32, $L = A_1 \oplus \langle 6 \rangle$, $M = A_1^* \oplus \langle 1/6 \rangle$ should be replaced by $L = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}$, $M = \frac{1}{6}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. For No.70, $M = (\boldsymbol{Z}/2\boldsymbol{Z})^2$ should read $M = \boldsymbol{Z}/4\boldsymbol{Z}$.

2. Existence theorem. The main result in this note is an existence theorem stating that all the 74 types actually occur (at least in case the ground field k has characteristic 0). More precisely, we can prove a much