61. *Mordell-Weil Lattices for Higher Genus Fibration*

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1. Introduction. The notion of the Mordell-Weil lattice of an elliptic curve over a function field (or of an elliptic surface) has been established in our previous work together with its basic properties (see [4], [5]). In this note, we sketch a generalization to the case of an algebraic curve of higher genus over a function field (or of an algebraic surface with higher genus fibration), and give a nontrivial example. Detailed account is in preparation.

Let $K = k(C)$ be the function field of an algebraic curve C over an algebraically closed ground field k; the curve C should serve as the base curve of some fibration and it is assumed to be smooth and projective. Let Γ/K be a smooth projective curve of genus $g > 0$ with a K-rational point $O \in \Gamma(K)$, and let J/K denote the Jacobian variety of Γ/K. Assume the following condition:

($*$) The K/k-trace of J is trivial.

Then the group of K-rational points $J(K)$ is a finitely generated abelian group (Mordell-Weil theorem), and the set $\Gamma(K)$ of K-rational points of Γ is a finite subset of $J(K)$ if $g > 1$ (Mordell conjecture for function fields = Theorem of Grauert-Manin-Samuel). We refer to Lang's book [2] for the above.

The main idea of this note is *to view the Mordell-Weil group $J(K)$ (modulo torsion) as a Euclidean lattice* with respect to a natural pairing defined in terms of intersection theory on an associated surface, in the same way as the case of $g = 1$ (cf. [4], [5]).

2. Basic theorems. Given Γ/K as above, we can associate an algebraic surface with a relatively minimal fibration:

$$f : S \to C. \tag{1}$$

Namely, S is a smooth projective surface, f is a morphism with the generic fibre Γ/K and there are no exceptional curves of the first kind in any fibre. The K-rational points of Γ are in a natural one-one correspondence with the sections of f; for $P \in \Gamma(K)$, (P) will denote the section regarded as a curve in S. Let $\mathrm{NS}(S)$ be the Néron-Severi group of S. Then we have

Theorem 2.1. *Under the assumption ($*$), there is a natural isomorphism:*

$$J(K) \simeq \mathrm{NS}(S)/T \tag{2}$$

where T is the subgroup generated by (O) and all the irreducible components of fibres of f.

For simplicity, assume in the following that ($**$) $\mathrm{NS}(S)$ is torsion-free. Then it forms an integral lattice with respect to the intersection pairing, of signature $(1, \rho - 1)$ (Hodge index theorem), $\rho = \mathrm{rk}\,\mathrm{NS}(S)$ being