# 4. On the Divisor Function and Class Numbers of Real Quadratic Fields. IV

By R. A. MOLLIN

Department of Mathematics and Statistics, University of Calgary

(Communicated by Shokichi IYANAGA, M. J. A., Jan. 13, 1992)

In this paper we conclude the investigation begun in [2]–[3] and [7]. We refer the reader to [2]–[3] for the notation and background material used herein.

Our first result generalizes Corollaries 2.1 and 2.2 of [7], (which we were only able to prove for ERD-types therein), and give, thereby, corrections to [4, Theorems 2.1–2.2, pp. 120–121]. First we deal with the case where $d \not\equiv 1 \pmod 4$.

**Theorem 1.** *Let $d=b^2+r\not\equiv 1 \pmod 4$ with $|r|<2b$ and $r$ odd. Set $A=(2b-|r-1|)/2$ and assume $P_d(A)\cap \mathcal{R}_I(d)=\{2, A\}$ where $I$ is the ideal over 2 and $P=\{$primes $p : p\,|\,A\}$. Thus*

$$h(d)\geq \tau(A).$$

*Proof.* Since $A<\sqrt{d}$ then $P_d(A)\cap Q_I(d)\subseteq P_d(A)\cap \mathcal{R}_I(d)$, and so the result now follows from Theorem 2.1 of [7].

**Remark 1.** The weaker hypothesis given in Theorem 2.1 of [4]; (viz., that no divisor $m$ of $(2a-|r-1|/4)$ with $1<m<(2a-|r-1|/4)$ appears in $\mathcal{R}_1(d)$), is insufficient to yield the conclusion therein, which is weaker than Theorem 2, below. For example if $d=385=20^2-15$ then $A=6$. Here $h(d)=2$ but $\tau(A)-1=3$. The problem is that $4\in \mathcal{R}_1(d)$. In fact any time that there is a divisor of $A^2$ (not just $A$) with $1<m<A$ with $m\in \mathcal{R}_1(d)$ then Theorem 2.2 of [4] fails to hold.

**Theorem 2.** *Let $d=b^2+r\equiv 1 \pmod 4$ with $|r|<2b$ and $r$ odd. Set $A=(2b-|r-1|)/4$, $P=\{$primes $p : p\,|\,A\}$ and assume $P_d(A)\cap \mathcal{R}_1(d)=\{1, A\}$ then*

$$h(d)\geq \tau(A)-2^n$$

*where $n=n(A)$.*

*Proof.* This follows from Theorem 2.1 of [7].

**Remark 2.** Corollary 2.2 of [7] is immediate from the above. Thus Theorem 1–2 correct [4, Theorems 2.1–2.2, pp. 120–121] for the cases where $r$ is odd. Now we look at the case where $r$ is even.

**Theorem 3.** *Let $d=b^2+r$ with $r$ even and $|r|<2b$ and set*

$$A=\begin{Bmatrix} 2b-|r-1| & \text{if } d\not\equiv 1 \pmod 4 \\ b-|r/4-1| & \text{if } d\equiv 1 \pmod 4 \end{Bmatrix}.$$

*Assume that if $m\,|\,A^2$ where $m>1$ is divisible by only unramified primes then $m\notin Q_1(d)$ (i.e., no such $m$ is the norm of a primitive principal ideal). Then with $n=n(A)$,*