28. *Retractive Nil-extensions of Regular Semigroups. I*

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Abstract: As retract extensions can be more easily constructed than many other kinds of extensions, it is of interest to know whether a given extension is a retract extension. The purpose of this paper is to give some criterions for retractive nil-extensions of semigroups, especially for the very important class of regular semigroups.

As retract extensions can be more easily constructed than many other kinds of extensions, it is of interest to know whether a given extension is a retract extension. It is known (see [5, p. 89]) that *every retract extension of a semigroup by a semigroup with zero is a subdirect product of these semigroups.* The converse of this assertion is given, here, by Lemma 1. This result is crucial in the proof of the main result of this paper which is given by Theorem 1: *A semigroup S is a retractive nil-extension of a regular semigroup K iff S is a subdirect product of K and a nil-semigroup.* For the related results see [2] and [3].

Throughout this paper, Z^+ will denote the set of all positive integers. Let us denote by $E(S)$ the set of all idempotents of a semigroup S. An element a of a semigroup S with zero 0 is *nilpotent* if there exists $n \in Z^+$ such that $a^n = 0$. A semigroup S is a *nil-semigroup* if all of elements of S are nilpotents. If $n \in Z^+$, then a semigroup S is *n-nilpotent* if $S^n = \{0\}$. An ideal extension S of a semigroup K is a *nil-extension* (*n-nilpotent extension*) of K if S/K is a nil-semigroup (n-nilpotent semigroup). A subsemigroup K of a semigroup S is a *retract* of S if there exists a homomorphism φ of S onto K such that $\varphi(a) = a$ for all $a \in K$. Such a homomorphism is called a *retraction.* An ideal extension S of K is a *retract extension* (or *retractive extension*) of K if K is a retract of S. A semigroup S is an *n-inflation* of a semigroup K if S is a $(n+1)$-nilpotent extension and a retractive extension of K.

For undefined notions and notations we refer to [1] and [5].

In the next considerations the following results will be used:

Proposition 1. [4] *Let τ and ρ be congruences on a semigroup S such that $r \subseteq \rho$. Then the relation ρ/τ defined on S/τ by*

$$(a\tau, b\tau) \in \rho/\tau \Leftrightarrow (a, b) \in \rho,$$

is a congruence and $(S/\tau)/(\rho/\tau) \cong S/\rho$.

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