77. *Weakly Compact Weighted Composition Operators on Certain Subspaces of* $C(X, E)$

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Let X be a compact Hausdorff space and E a complex Banach space. By $C(X, E)$ we denote the Banach space of all continuous E-valued functions on X with the supremum norm. The compact weighted composition operators on $C(X, E)$ have been characterized by Jamison and Rajagopalan [2]. One of the authors proved an analogue for those operators on a more general space $A(X, E)$ ([6]). In this note, we characterize the weakly compact weighted composition operators on $A(X, E)$, and give some remarks on the difference between compactness and weak compactness of weighted composition operators.

Let A be a function algebra on X, that is, a uniformly closed subalgebra of $C(X)=C(X, \boldsymbol{C})$ which contains the constants and separates the points of X. We define the closed subspace $A(X, E)$ of $C(X, E)$ by

$$A(X, E)=\{f \in C(X, E) : e^* \circ f \in A \text{ for all } e^* \in E^*\},$$

where E^* is the dual space of E. We recall that a *weighted composition operator* of $A(X, E)$ is a bounded linear operator T from $A(X, E)$ into itself, which has the form;

$$Tf(x)=w(x)f(\varphi(x)), \qquad x \in X,\ f \in A(X, E),$$

for some selfmap φ of X and some map w from X into $B(E)$, the space of bounded linear operators on E. In the sequel, we write wC_φ in place of T. For a weighted composition operator wC_φ on $A(X, E)$, we have that $|||w|||=\sup\{\|w(x)\|_{B(E)} : x \in X\}<+\infty$, and that the map $w : X \to B(E)$ is continuous in the strong operator topology. We also know that φ is continuous on an open set $S(w)=\{x \in X : w(x) \neq O\}$ in X. Since X is imbedded into the maximal ideal space M_A of A, we sometimes consider the selfmap φ of X as a map from X into M_A. Notice that M_A is decomposed into (Gleason) parts for A. If every non-trivial part P satisfies the condition below, then the associated space $A(X, E)$ is said to have the *property* (α);

for any $x \in P$, *there are an open neighborhood* V *of* x *relative to* P *and a homeomorphism* ρ *from a polydisc* D^N *(*N *depends on* x*) onto* V *such that* $\hat{f} \circ \rho$ *is analytic on* D^N *for the Gelfand transform* $\hat{f}$ *of any* $f \in A$ (see [3]).

Simple examples of $A(X, E)$ with the property (α) are $C(X, E)$ and $\{f \in C(\bar{D}, E) : f$ is an analytic E-valued function on the interior of $\bar{D}\}$, where $\bar{D}$ is the closed unit disc. As a matter of notational convenience, we put $E_0=\{e \in E : \|e\|_E \leq 1\}$, and $E_0^*=\{e^* \in E^* : \|e^*\| \leq 1\}$. In what follows, we under-