75. On the Equation Describing the Random Motion of Mutually Reflecting Molecules

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Introduction. In this paper we construct a model for the random motion of M molecules mutually reflecting in $\boldsymbol{R}^d$ and investigate its limiting behavior as the size R of the molecules tends to 0. We assume that the k-th molecule consists of n_k (≥ 1) atoms. The atoms move randomly in the way as described below (see (0.3)) under the following restrictions (0.1) and (0.2).

(0.1) Any two atoms in different molecules reflect each other when the distance between them equals a given constant ρ (>0).

(0.2) The distance between any two atoms in the same molecule does not exceed a given constant R (>0).

Let $\Lambda=\{1, \cdots, N\}$, $N=\sum_{k=1}^{M} n_k$ and $\Lambda_k=\{\sum_{i=1}^{k-1} n_i+1, \sum_{i=1}^{k-1} n_i+2, \cdots, \sum_{i=1}^{k} n_i\}$, $k=1, \cdots, M$, where the convention $\sum_{i=1}^{0}=0$ is used. Λ_k describes the set of indexes of atoms in the k-th molecule. For each $i \in \Lambda$, we put $m(i)=k$ if $i \in \Lambda_k$. Denote by $X_i(t)$ the position of the i-th atom at time t and put $R_i(t) =\max_{j:m(j)=m(i)} |X_i(t)-X_j(t)|$, $\rho_i(t)=\min_{j:m(j)\neq m(i)} |X_i(t)-X_j(t)|$. We assume that the random motion of the atoms is described by the stochastic differential equation (SDE)

$$dX_i(t)=dB_i(t)+dL_i(t), \quad i=1, 2, \cdots, N, \tag{0.3}$$

where $B_i(t)$, $1\leq i\leq N$, are independent d-dimensional Brownian motions and each $L_i(t)$ is a process of bounded variation which can vary only when either $\rho_i(t)=\rho$ or $R_i(t)=R$ and represents effects of (0.1) and (0.2) so that $\rho_i(t)\geq\rho$, $R_i(t)\leq R$, $t\geq 0$, $1\leq i\leq N$. It is assumed that $\rho_i(t)\geq\rho$, $R_i(t)\leq R$, $t\geq 0$, $1\leq i\leq N$. Similar random motions were considered in [3] and [4]; however, in [3] the restriction (0.2) was not considered and in [4] $n_k=1$ for all k.

To solve (0.3) we consider the following problem. For given $\boldsymbol{w}=(w_1, w_2, \cdots, w_N) \in C([0, \infty)\to\boldsymbol{R}^{Nd})$ satisfying

$$\begin{aligned} |w_i(0)-w_j(0)| &\leq R \quad \text{for all} \quad i, j \quad \text{with} \quad m(i)=m(j), \\ &\geq \rho \quad \text{for all} \quad i, j \quad \text{with} \quad m(i)\neq m(j), \end{aligned}$$

we want to find $\xi_i^R(t)$ satisfying the equation

$$\xi_i^R(t)=w_i(t)+\sum_{\substack{j=1\\(\neq i)}}^{N} \int_0^t (\xi_i^R(s)-\xi_j^R(s))dl_{ij}^R(s) \quad i=1, 2, \cdots, N, \tag{0.4}$$

under the following conditions (i) and (ii):

(i) $\xi^R=(\xi_1^R, \xi_2^R, \cdots, \xi_N^R) \in C([0, \infty)\to\boldsymbol{R}^{Nd})$ with