47. *On the Cauchy-Kowalevskaya Theorem for Systems*[†]

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(Communicated by Kunihiko KODAIRA, M. J. A., June 11, 1991)

§1. **Introduction and result.** In this note, we shall study the Cauchy-Kowalevskaya theorem for systems of partial differential equations which are nondegenerate on ∂_t. (We say that a system is "nondegenerate on ∂_t" when it satisfies the major premise in Theorem 3.1 in M. Miyake [5].) Let Ω be an open set in $C_t^1 \times C_x^l$. If we require the Cauchy problem is uniquely solvable in $C[[t, x]]$, by virtue of Theorem 3.1 in M. Miyake [5], it is enough to consider the following Cauchy problem:

$$(1.1)\qquad \begin{cases} \partial_t u_i(t,x) - \sum_{j=1}^{N} a_{ij}(t,x,\partial_x)u_j(t,x) = f_i(t,x), \\ u_i(t_\circ, x) = \varphi_i(x), \qquad (1 \le i \le N), \end{cases}$$

where all coefficients are holomorphic in Ω, f_i $(1 \le i \le N)$ are given holomorphic functions, φ_i $(1 \le i \le N)$ are holomorphic initial data and u_i $(1 \le i \le N)$ are unknown functions.

We also denote (1.1) by

$$(1.1')\qquad \begin{cases} P(t,x,D_t,D_x)u \equiv D_t u(t,x) - A(t,x,D_x)u(t,x) = f(t,x), \\ \qquad u(t_\circ, x) = \varphi(x), \end{cases}$$

where D_t and D_x are $(\sqrt{-1})^{-1}\partial_t$ and $(\sqrt{-1})^{-1}\partial_x$ respectively, $A(t,x,D_x)$ is an $N \times N$ matrix and u, f and φ are N-vectors.

We say that the Cauchy-Kowalevskaya theorem holds for $P(t,x,D_t,D_x)$ when, for any $(t_\circ, x_\circ)$ in Ω, any neighborhood ω of $(t_\circ, x_\circ)$, any $f(t,x)$ in $\mathcal{H}(\omega)$ and any $\varphi(x)$ in $\mathcal{H}(\omega \cap \{t = t_\circ\})$, there exists an unique holomorphic solution $u(t,x)$ of (1.1′) in a neighborhood of $(t_\circ, x_\circ)$. Here, $f(t,x) \in \mathcal{H}(\omega)$ means that $f(t,x)$ is holomorphic in ω, and so on.

When the order of $A(t,x,D_x)$ is at most one, the Cauchy-Kowalevskaya theorem holds. So, we are interested in the case that the order ν of $A(t,x,D_x)$ is greater than one.

In the case of constant coefficients, the necessary and sufficient condition for the Cauchy-Kowalevskaya theorem is that the characteristic polynomial $\det(\tau I - A(t,x,\zeta))$ is a Kowalevskian polynomial, that is, its degree on τ and ζ is at most N. (See S. Mizohata [6].) As was clarified in [6], the above condition is neither necessary nor sufficient in the case of variable coefficients. In [6], S. Mizohata proposed a necessary condition. Following it, M. Miyake obtained the necessary and sufficient condition in case

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†) Dedicated to Professor Shigetake MATSUURA on his sixtieth birthday.

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