33. On a Remarkable Class of Homogeneous Symplectic Manifolds

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In this note, we present some results*) on homogeneous symplectic manifolds M admitting a pair of transversal Lagrangian foliations (The class of these manifolds contains parahermitian symmetric spaces introduced and studied in [1], [2]). To such a manifold M we associate an algebraic object, called a (weak) dipolarization in a Lie algebra. We construct a natural compactification of such a manifold M arising from a semisimple graded Lie algebra. Also we give the infinitesimal classification of such manifolds corresponding to simple graded Lie algebras. The details will appear elsewhere.

1. Let M be a (connected) symplectic manifold with symplectic form ω, and let (F^+, F^-) be a pair of transversal completely integrable distributions on M. Then the triple $(M, \omega, F^{\pm})$ (or simply M) is said to be a *parakähler manifold* if each leaf of $F^{\pm}$ is a Lagrangian submanifold of M. A parakähler manifold is originally introduced by P. Libermann [4] by a different point of view (see also [1]). Let $(M, \omega, F^{\pm})$ be a parakähler manifold. By an *automorphism* of M we mean a symplectomorphism of M which leaves the distributions $F^{\pm}$ invariant. We denote by $\mathrm{Aut}\, M$ the full group of automorphisms of M, which turns out to be a finite-dimensional Lie group. If $\mathrm{Aut}\, M$ acts transitively on M, then M is called a *homogeneous* parakähler manifold. Let G be a connected Lie group and H be a closed subgroup of G. If the coset space G/H admits a parakähler structure $(\omega, F^{\pm})$ and if G acts on G/H as automorphisms, then we say that the parakähler structure $(\omega, F^{\pm})$ is *G-invariant* and that G/H is a *parakähler coset space*. A homogeneous parakähler manifold may be expressed as various parakähler coset spaces. In our situation we can consider a "parakähler algebra" which is an analogue to a Kähler algebra (Vinberg-Gindikin [5]) for a homogeneous Kähler manifold.

Definition 1. Let $\mathfrak{g}$ be a real Lie algebra, $\mathfrak{g}^{\pm}$ be two subalgebras of $\mathfrak{g}$ and ρ be an alternating 2-form on $\mathfrak{g}$. The triple $\{\mathfrak{g}^+, \mathfrak{g}^-, \rho\}$ is called a *weak dipolarization* in $\mathfrak{g}$, if the following conditions are satisfied:

WD1) $\mathfrak{g} = \mathfrak{g}^+ + \mathfrak{g}^-$,

WD2) Put $\mathfrak{h} := \mathfrak{g}^+ \cap \mathfrak{g}^-$. Then $\rho(X, \mathfrak{g}) = 0$ if and only if $X \in \mathfrak{h}$,

WD3) $\rho(\mathfrak{g}^+, \mathfrak{g}^+) = \rho(\mathfrak{g}^-, \mathfrak{g}^-) = 0$,

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