27. *Fermat Motives and the Artin-Tate Formula. I*

By Noriyuki SUWA
Department of Mathematics, Tokyo Denki University

(Communicated by Shokichi IYANAGA, M. J. A., April 12, 1991)

In this note, we mention some results on the Artin-Tate formula for Fermat motives in the higher dimensional cases, which was achieved by [4] and [10] in the 2-dimensional case. Detailed account will be published elsewhere.

1. Definition of Fermat motives (Shioda [4]). **1.1.** Let k be a field and let X be the Fermat variety of dimension n and of degree m over k:

$$X: T_0^m + T_1^m + \cdots + T_{n+1}^m = 0 \subset \boldsymbol{P}_k^{n+1}.$$

We assume that $(m, p)=1$ if k is of characteristic $p>0$. Let μ_m denote the group of m-th root of unity in $\bar{k}$. The group $G=(\mu_m)^{n+2}/(\text{diagonal})$ acts naturally on $X_{\bar{k}} = X \otimes_k \bar{k}$. The character group $\hat{G}$ of G is identified with the set

$$\left\{\boldsymbol{a}=(a_0, a_1, \cdots, a_{n+1})\,;\ a_i \in \boldsymbol{Z}/m,\ \sum_{i=0}^{n+1} a_i = 0\right\};$$

Let $(\boldsymbol{Z}/m)^\times$ act on $\hat{G}$ by $t\boldsymbol{a}=(ta_0, \cdots, ta_{n+1}) \in \hat{G}$ for any $\boldsymbol{a} \in \hat{G}$ and $t \in (\boldsymbol{Z}/m)^\times$.

Let ζ_m be a fixed primitive m-th root of unity in $\bar{\boldsymbol{Q}}$. For the $(\boldsymbol{Z}/m)^\times$-orbit A of $\boldsymbol{a}=(a_0, \cdots, a_{n+1}) \in \hat{G}$, define

$$p_A = \frac{1}{m^{n+1}} \sum_{g \in G} \mathrm{Tr}_{\boldsymbol{Q}(\zeta_m^d)/\boldsymbol{Q}}(\boldsymbol{a}(g)^{-1}) g \in \boldsymbol{Z}\left[\frac{1}{m}\right][G].$$

Here $d=\gcd(m, a_0, \cdots, a_{n+1})$. Then p_A are idempotents, i.e.

$$p_A \cdot p_B = \begin{cases} p_A & \text{if } A=B \\ 0 & \text{if } A \neq B \end{cases}, \quad \sum_{A \in O(\hat{G})} p_A = 1$$

where $O(\hat{G})$ denotes the set of $(\boldsymbol{Z}/m)^\times$-orbits in $\hat{G}$. The pair $M_A=(X, p_A)$ defines a motive over k, called the *Fermat submotive* of X corresponding to A (Shioda [4], p. 125).

1.2. Define a subset $\mathfrak{A}$ of $\hat{G}$ by

$$\mathfrak{A} = \{\boldsymbol{a}=(a_0, \cdots, a_{n+1}) \in \hat{G}\,;\ a_i \neq 0 \text{ for all } i\}.$$

For each $\boldsymbol{a} \in \mathfrak{A}$, let

$$\|\boldsymbol{a}\| = \sum_{i=1}^{n+1} \left\langle \frac{a_i}{m} \right\rangle - 1$$

where $\langle x \rangle$ stands for the fractional part of $x \in \boldsymbol{Q}/\boldsymbol{Z}$.

1.3. Let R be a ring, in which m is invertible, and let F be a contravariant functor from a category of varieties over k to the category of R-modules. For a Fermat submotive $M_A=(X, p_A)$ of X, define

$$F(M_A) = \mathrm{Im}\,[p_A^* : F(X) \to F(X)].$$

Example **1.4.** Let l be prime number different from the characteristic of k. The l-adic étale cohomology groups $H^{\cdot}(X, \boldsymbol{Q}_l(i))$, $i \in \boldsymbol{Z}$; moreover, if l