26. *Chiral Anomaly on a Spin Manifold*

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1. **The aim.** We propose a rigorous framework to deduce the chiral anomaly on a compact spin manifold: we show that a certain functional integral describing the second quantized fermion gives this anomaly.

To this end, based on triangulations of the manifold, we define finite dimensional approximations (the lattice regularization) of the functional integral and take the continuum limit. The limit is shown to be a topological invariant by means of the index theorem.

This work is the continuation of [1], which deals only with the case of a flat torus. In the present work, we improve the formulation in [1] by substituting both the Wilson term and the regularizing heat kernel by a single operator in the action. See [2] for details.

2. **Chiral anomaly.** Let M be a compact oriented Riemannian spin manifold of dimension d ($=$even) and P a principal bundle over M with group G. Let ρ be a unitary representation of G on $\boldsymbol{C}^N$ giving the associated vector bundle $E=P\times_G \boldsymbol{C}^N$ with a Hermitian inner product $\langle\ ,\ \rangle$. We now fix a connection A of P and denote by $\boldsymbol{D}$ the Dirac operator defined by A. $\boldsymbol{D}$ acts on $\Gamma(S\otimes E)$, the space of all smooth sections of $S\otimes E$, where S is the spinor bundle over M.

Let e_μ ($\mu=1,\cdots,d$) be orthonormal vector fields around x. Let $J^\mu(x)$ be the chiral current $\langle\psi(x),\ \tau e_\mu\psi(x)\rangle$, where $\psi\in\Gamma(S\otimes E)$, $\tau=i^{d/2}e_1e_2\cdots e_d$ and products mean the Clifford multiplications. Then, one has a vector field J on M given by

$$J(x)=\sum_{\mu=1}^{d} J^\mu(x)e_\mu(x).$$

A solution $\psi(x)$ to the Dirac equation $\boldsymbol{D}\psi(x)=0$ is subject to the conservation law:

$$\operatorname{div} J(x)=0. \tag{1}$$

The chiral anomaly is the breakdown of the conservation law (1) after the second quantization: according to physical literatures (see e.g. [3]), *the vacuum expectation* $\langle\operatorname{div} J\rangle$ in general does not vanish, though the field operator $\psi(x)$ satisfies $\boldsymbol{D}\psi(x)=0$. Moreover, the vacuum expectation of

$$Y(x)=\operatorname{div} J(x)-2im\langle\psi(x),\ \tau\psi(x)\rangle$$

can be identified with the characteristic class that appears in the index formula [4], where $m>0$ is the fermion mass. Our problems are to define and

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