26. The Modulo 2 Homology Group of the Space of Rational Functions

By Yasuhiko KAMIYAMA
Department of Mathematics, Faculty of Science, University of Tokyo

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§1. **Introduction.** Let $\hat{M}_k$ be the moduli space of $SU(2)$ monopoles associated with Yang-Mills-Higgs and Bogomol'nyi equations. It is shown [1] that $\hat{M}_k$ is homeomorphic to the space of based holomorphic maps of degree k from S^2 to S^2.

More generally we define $F_k^*(S^2, CP^m)$ to be the space of based holomorphic maps of degree k from S^2 to CP^m.

Segal [3] studied the connection between $F_k^*(S^2, CP^m)$ and $\Omega_k^2 CP^m$. The result is as follows

Theorem 1 [3]. *The inclusion*

$$i: F_k^*(S^2, CP^m) \longrightarrow \Omega_k^2 CP^m$$

is a homotopy equivalence up to dimension $k(2m-1)$, *the induced homomorphism* $i_*: \pi_q(F_k^*(S^2, CP^m)) \to \pi_q(\Omega_k^2 CP^m)$ *is bijective for* $q < k(2m-1)$ *and surjective for* $q = k(2m-1)$.

It is well know [2] that $\coprod_k \Omega_k^2 CP^m$ has natural loop sum and C_2 structure.

Recently Boyer and Mann [1] introduced loop sum and C_2 structure in $\coprod_k F_k^*(S^2, CP^m)$ which are compatible with the inclusion i. Hence we can naturally define the loop sum and the Dyer-Lashof operation Q_1 in $\oplus_k H_* (F_k^*(S^2, CP^m); Z_2)$.

By using these methods, Boyer and Mann produced certain elements in $H_*(F_k^*(S^2, CP^m); Z_2)$ some of which have degree greater than $k(2m-1)$. (cf. Theorem 1.)

Then the following question arises naturally.

Question. Are the elements produced by the loop sum and the Dyer-Lashof operation from ι_{2m-1} (ι_{2m-1} will be defined later) the basis of $H_*(F_k^*(S^2, CP^m); Z_2)$?

We shall study this question. The results are as follows. We write F_k^* for $F_k^*(S^2, CP^1)$.

Theorem A. *The elements produced by the loop sum and the Dyer-Lashof operation from* ι_1 *are the basis of* $H_*(F_2^*; Z_2)$.

Theorem B. *For* $m \geqq 2$, *the elements produced by the loop sum and the Dyer-Lashof operation from* ι_{2m-1} *are the basis of* $H_*(F_2^*(S^2, CP^m); Z_2)$.

Theorem C. *For* $m \geqq 2$, *the elements produced by the loop sum and the Dyer-Lashof operation from* ι_{2m-1} *are the basis of* $H_*(F_3^*(S^2, CP^m); Z_2)$.

Theorem D. *For* $m \geqq k+1$, *the elements produced by the loop sum*