21. Nonlinear Singular First Order Partial Differential Equations of Briot-Bouquet Type

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In this paper we will present a generalization of the Briot-Bouquet ordinary differential equation to partial differential equations.

§1. **Briot-Bouquet equation.** First let us recall the theory of nonlinear ordinary differential equations of the form

$$t\frac{du}{dt}=f(t,u),\quad f(0,0)=0 \tag{1.1}$$

which was first studied by Briot-Bouquet [1]. Nowadays it is called the Briot-Bouquet equation and the structure of solutions of (1.1) near the origin of C_t is well-known (see Hille [3], Hukuhara-Kimura-Matuda [4], Kimura [5], Gérard [2] etc.). In particular, when

$$\rho=\frac{\partial f}{\partial u}(0,0)$$

is in a generic position, we know the following:

Theorem 1. *Assume that $f(t,u)$ is a holomorphic function defined near the origin of $C_t\times C_u$. Then we have:*

(1) (*Holomorphic solutions*). *If $\rho\notin N^*(=\{1,2,3,\cdots\})$, the equation* (1.1) *has a unique solution $u_0(t)$ holomorphic near the origin of C_t satisfying $u_0(0)=0$.*

(2) (*Singular solutions*). *If $\rho\notin N^*\cup\{a\in R;\ a\leqq 0\}$, the general solution $u(t)$ of* (1.1) *near the origin of C_t is given by*

$$u(t)=ct^{\rho}+a_{1,0}t+\sum_{i+j\geqq 2}a_{i,j}t^i(ct^{\rho})^j, \tag{1.2}$$

where $c\in C$ is arbitrary, the coefficients $a_{i,j}\in C$ are uniquely determined by the equation (1.1), *and the series*

$$w+a_{1,0}t+\sum_{i+j\geqq 2}a_{i,j}t^iw^j$$

is a convergent power series in $\{t,w\}$. The holomorphic solution $u_0(t)$ in (1) *is given by the case $c=0$.*

§2. **Generalization of (1.1) to partial differential equations.** Let us consider

$$t\frac{\partial u}{\partial t}=F\left(t,x,u,\frac{\partial u}{\partial x}\right), \tag{2.1}$$

where $(t,x)\in C_t\times C_x^n$, $x=(x_1,\cdots,x_n)$, $\dfrac{\partial u}{\partial x}=\left(\dfrac{\partial u}{\partial x_1},\cdots,\dfrac{\partial u}{\partial x_n}\right)$ and $F(t,x,u,v)$ with $v=(v_1,\cdots,v_n)$ is a function defined in a polydisk Δ centered at the

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