19. *Certain Quadratic First Integral and Elliptic Orbits of Linear Hamiltonian System*

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1. Introduction. This paper deals with a close relation between a hyperplane filled with elliptic orbits of a linear Hamiltonian system and a certain quadratic first integral. To be more precise, it is proved that when a linear Hamiltonian system admits an invariant hyperplane filled with closed orbits, it leaves a quadratic form invariant, and conversely, when a certain quadratic first integral is admitted, there exists such an invariant hyperplane.

By the way, the phase portrait drawn by a discrete-time system which approximates a continuous Hamiltonian system is often different from that of the original system. For example, a closed orbit of the original system is usually destroyed by a discrete system, even when the original one is linear. It seems that the result of this paper is of use for the purpose of reproducing the original elliptic orbit by a discrete system when a certain kind of first integrals is inherited.

2. Elliptic orbit of linear system. Let us think of a linear Hamiltonian system with N degrees of freedom given by

$$\frac{dx}{dt}=Hx, \quad H\in sp(N,R), \quad x\in R^{2N}. \tag{1}$$

We introduce into the phase space R^{2N} both a Euclidean inner product $(x,y)={}^t xy$ and a symplectic inner product $\langle x,y\rangle={}^t xJy$, where $J=\begin{bmatrix}0 & I\\ -I & 0\end{bmatrix}$ and the superfix t denotes matrix transpose. An orbit of (1) which starts from x_0 is closed, it and only if $e^{\varepsilon H}x_0=x_0$ holds for a positive constant ε. This condition is equivalent to that H has pure imaginary eigenvalues, in other words, H^2 has a negative eigenvalue. Then, we define a linear subspace by

$$\Gamma_\beta=\{x\in R^{2N}\,|\,H^2x=-\beta^2x\} \qquad (\beta>0), \tag{2}$$

and assume that $\Gamma_\beta\neq\{0\}$ from now on. Let us pay attention to the solution curves of (1) which are contained in Γ_β. Choose an arbitrary $q\in H_\beta$, $q\neq 0$, and put

$$p=-\frac{1}{\beta}Hq. \tag{3}$$

Then, q and p are linearly independent and spans a two-dimensional hyperplane Γ included by Γ_β.

Proposition 1. *The orbit of* (1) *starting from* $q\in\Gamma$ *is an ellipse with the period* $2\pi/\beta$, *and lies in* Γ. *Furthermore, all elliptic orbits in* Γ *are*