16. *A Table of the Dimensions of the Extended Hilbert Modular Type Cusp Forms*

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1. Introduction and the table. For a square-free positive number D, let k be a real quadratic number field $\boldsymbol{Q}(\sqrt{D})$. Let $\mathfrak{o}$, U and U^+ be the ring of integers in k, the group of units in $\mathfrak{o}$ and the group of all totally positive units. The extended Hilbert modular group is defined as follows

(1) $$\hat{\Gamma}=\{\gamma \in GL_2(\mathfrak{o})\,;\, \det(\gamma) \in U^+\}\Big/\left\{\begin{bmatrix}\varepsilon, 0\\ 0, \varepsilon\end{bmatrix};\ \varepsilon \in U\right\}.$$

Hausmann investigated the fixed points of $\hat{\Gamma}$ in [1]. When k has a unit of negative norm, $\hat{\Gamma}$ coincides with the ordinary Hilbert modular group Γ. We consider the space $\hat{S}(D)$ of the cusp forms of weight two with respect to $\hat{\Gamma}$ in H^2 (H being a complex upper half plane).

For the ordinary Hilbert modular group, we have already given a dimension table in [5] of which this note is a continuation. We tabulate the dimension of $\hat{S}(D)$ for a square-free D and $1<D<1000$. In the following table, the number D is given by

(2) $$D=i+j \qquad (i=\text{row number},\ j=\text{column number}).$$

When the mark '$-$' appears after a figure, $\boldsymbol{Q}(\sqrt{D})$ has a unit of negative norm. The mark '$**$' means that D is not square-free. To calculate this table, we used ACOS-6 computer system in Okayama University Computer center.

2. The method of the computation. From now on, we will only treat with the case of $\hat{\Gamma}\neq\Gamma$. For a square-free divisor w of the discriminant d_k of k, let Γ_w be the subgroup of $PL_2(k)$ generated by Γ and the set of elements $\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ (mod k^X) such that $a,\ b,\ c,\ d \in (w)^{1/2}$, $ad-bc=w$, where $(w)^{1/2}$ is an ideal whose square equals (w). When $\tilde{w}$ is a square-free part of d_k/w, $\Gamma_w=\Gamma_{\tilde{w}}$. There exists some w such that $\Gamma_w=\hat{\Gamma}$.

By virtue of [1], [3], we get

Theorem. *Let w be a divisor of d_k satisfying $\Gamma_w=\hat{\Gamma}$. The dimension of $\hat{S}(D)$ is given by*

(3) $$\dim \hat{S}(D)=t_0+t_1+t_2-1$$

Each term can be written as follows.

(4) $$t_0=(1/4)\zeta_k(-1)$$

(5) $$t_1=a(D, w)h(-D)+b(D, w)h(-3D)+c(D, w)h(-w)h(-\tilde{w})$$