80. *An Application of a Certain Fractional Derivative Operator*

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The object of the present paper is to introduce and study a linear operator $\mathcal{J}_{0,z}^{\alpha,\beta,\eta}$ which is defined in terms of a certain fractional derivative operator. Various interesting properties of the operator $\mathcal{J}_{0,z}^{\alpha,\beta,\eta}$, including its connection with the Carlson-Shaffer operator $\mathcal{L}(a, c)$, are given. It is also shown how these operators can be applied successfully with a view to proving a number of inclusion and connection theorems involving starlike, convex, and prestarlike functions in the open unit disk $\mathcal{U}$.

1. **Introduction.** Let $\mathcal{A}$ be the class of functions of the form:

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the *open* unit disk

$$\mathcal{U}=\{z: |z|<1\}.$$

A function $f(z) \in \mathcal{A}$ is said to be *starlike of order* α if it satisfies the inequality:

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha \tag{1.2}$$

for some α $(0\leqq\alpha<1)$ and for all $z \in \mathcal{U}$. We denote by $\mathcal{S}^*(\alpha)$ the subclass of $\mathcal{A}$ consisting of functions which are starlike of order α.

Furthermore, a function $f(z) \in \mathcal{A}$ is said to be *convex of order* α if it satisfies the inequality:

$$\operatorname{Re}\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\alpha \tag{1.3}$$

for some α $(0\leqq\alpha<1)$ and for all $z \in \mathcal{U}$. We denote by $\mathcal{K}(\alpha)$ the subclass of $\mathcal{A}$ consisting of all functions which are convex of order α.

Throughout this paper, it should be understood that functions such as

$$\frac{zf'(z)}{f(z)} \quad\text{and}\quad \frac{zf''(z)}{f'(z)},$$

which have *removable singularities* at $z=0$, have had these singularities removed in statements like (1.2) and (1.3).

It follows readily from (1.2) and (1.3) that (cf. Duren [2, p. 43, Theorem 2.12] for the special case $\alpha=0$)

$$f(z) \in \mathcal{K}(\alpha) \Longleftrightarrow zf'(z) \in \mathcal{S}^*(\alpha). \tag{1.4}$$

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