55. *The Mean Square of Dirichlet L-functions*

(*A Generalization of Balasubramanian's Method*)

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§ 1. The present article is a corrected version of the author's erroneous note [4]. (The report [5] includes the same errors. The corrections of the results have already been published in [6].) Let χ be a primitive Dirichlet character mod q, and $L(s, \chi)$ the corresponding Dirichlet L-function. Then,

Theorem. *For any odd* q *and any* $T>0$, *we have*

$$\int_0^T \left|L\left(\frac{1}{2}+it, \chi\right)\right|^2 dt=\frac{\varphi(q)}{q} T \log T+\frac{\varphi(q)}{q}\left(\log \frac{q}{2\pi}+2\gamma-1+2\sum_{p|q}\frac{\log p}{p-1}\right)T +E(T, \chi),$$

where $\varphi(q)$ *is Euler's function,* γ *is Euler's constant,* p *runs over all prime divisors of* q, *and*

$$(1.1)\quad E(T, \chi)=O((qT)^{1/3+\varepsilon}+q(\log q)(qT)^{1/4}(\log (T+1))^{1/2}+q^{7/4}(\log q)(qT)^{1/4} +q^{21/8}(\log q)(qT)^{1/8}+q^{13/4}(\log q)(\log q(T+1))^2+q^5(\log q)(\log q(T+1)))$$

for any $\varepsilon>0$.

In particular, if $T\gg q^{20}$, then $E(T, \chi)\ll(qT)^{1/3+\varepsilon}$. Motohashi [9] gives a better estimate in case q is a prime.

Only after the appearance of Motohashi's above mentioned work and the announcement (in Zentralblatt für Mathematik) of Meurman's paper [8], the author had noticed that the statement in [4]* is incorrect. Then the author started checking the former calculations, and found two essential errors. One of them is related with the Riemann-Siegel formula of L-functions (see (2) in [6]). Using the notations in [4], we can state it as

$$(1.2)\qquad e^{i\vartheta}L\left(\frac{1}{2}+it, \chi\right)=f_1(t)+f_2(t)+f_3(t),$$

and the correct estimate of the error term $f_3(t)$ is $O(q^{5/4}t^{-3/4})$. From (1.2), we have

$$(1.3)\quad \int_0^T \left|L\left(\frac{1}{2}+it, \chi\right)\right|^2 dt=\int_0^T f_1(t)^2dt+\int_0^T f_2(t)^2dt+\int_0^T f_3(t)^2dt +2\int_0^T f_1(t)f_2(t)dt+2\int_0^T f_2(t)f_3(t)dt+2\int_0^T f_3(t)f_1(t)dt.$$

The other error in [4] is in the evaluation of the fourth integral in the right-hand side of (1.3). It is the reason of the appearance of the term $4(2\pi/q^3)^{1/2}(\Sigma' n)T^{1/2}$ in Theorem 1 of [4], which is to be omitted. A drastic change of the argument was required to correct this error, and the