46. *Newforms of Half-integral Weight and the Twisting Operators*

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0. In the papers [4] and [5], we report some trace relations of the twisting operators on the space of cusp forms of half-integral weight $S(k+1/2, N, \chi)$ and on the Kohnen subspace $S(k+1/2, N, \chi)_K$. In this paper, we shall use these trace relations of the twisting operators in order to decompose the spaces $S(k+1/2, N, \chi)$ and $S(k+1/2, N, \chi)_K$ into nice subspaces, i.e., the space of "*newforms*" which correspond in one to one way to a system of eigen-values for Hecke operators. For simplicity of statements, we treat only the case of the Kohnen subspace of level $4p^m$, weight $k+1/2$ and a character χ, where p is an odd prime number, $2\leq m \in \boldsymbol{Z}$, $2\leq k \in \boldsymbol{Z}$, and χ is an even character modulo $4p^m$ such that $\chi^2=1$. More general results and details will appear in [6].

1. We keep to the notations and the assumptions in [4]. Let $\psi=\left(\frac{}{p}\right)$ be the quadratic residue symbol. Since the twisting operator R_ψ for ψ satisfies the identity $R_\psi^3=R_\psi$ as operators, R_ψ is a semi-simple operator and the eigen values of R_ψ are 1, 0, or -1. We denote the σ-eigen subspace of $\tilde{S}=\tilde{S}(p^m, \chi)=S(k+1/2, 4p^m, \chi)_K$, $\sigma=0$, 1, or -1, by: $\tilde{S}^0=\tilde{S}^0(p^m, \chi)$ if $\sigma=0$ and $\tilde{S}^\pm=\tilde{S}^\pm(p^m, \chi)$ if $\sigma=\pm 1$. Then we have $\tilde{S}=\tilde{S}^0\oplus\tilde{S}^+\oplus\tilde{S}^-$ and moreover

$$\tilde{S}^0=\mathrm{Ker}\,(R_\psi\,|\,\tilde{S})=\left[S\left(k+1/2, 4p^{m-1}, \chi\left(\frac{p}{}\right)\right)_K\right]^{(p)}.$$

Here, we put $[S(k+1/2, 4p^m, \chi)_K]^{(p)}=\{f(pz)\,|\,f\in S(k+1/2, 4p^m, \chi)_K\}$. This equality follows from the following lemma.

Lemma. *Let N be a positive integer divisible by 4, χ an even character modulo N, and l an odd prime divisor of N. If a function f on $\mathfrak{H}$ is satisfies the following two conditions*:

(i) $f(z)=f(z+1)$ *for all* $z\in\mathfrak{H}$, (ii) $f(lz)\in S(k+1/2, N, \chi)$,
then we have

$$f\in S\left(k+1/2, N/l, \chi\left(\frac{l}{}\right)\right).$$

In particular, if the conductor of $\chi\left(\frac{l}{}\right)$ does not divide N/l, then $f=0$.

Remark. This lemma is an analogy of the Theorem 4.6.4 of [1].

From $\tilde{T}(n^2)R_\psi=R_\psi\tilde{T}(n^2)$ ([5, Prop. (1.7)]), we have the following formulae: