42. *Deformations of Complex Analytic Subspaces with Locally Stable Parametrizations of Compact Complex Manifolds*

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Introduction. In this paper we shall give a definition of *complex analytic subspaces with locally stable parametrizations* of compact complex manifolds, which is a generalization of closed complex analytic subsets of *simple normal crossing* in [3] and analytic subvarieties with *ordinary singularities* in [8], and we show that their *logarithmic deformations* and *locally trivial displacements* are equivalent to deformations of *locally stable holomorphic maps* (cf. Definition 1.1 below). From this equivalence and Miyajima-Namba-Flenner's theorem on the existence of the Kuranishi family of deformations of holomorphic maps, it follows that there exist the Kuranishi family of logarithmic deformations and the maximal family of locally trivial displacements of a complex analytic subspace with a locally stable parametrization. These are a unification and a generalization of the results in [3] and [8]. Throughout this paper all complex analytic spaces are assumed to be reduced, second countable, and finite dimensional. For notation and terminology concerning logarithmic deformations, locally trivial displacements of a complex analytic subspace and deformations of a holomorphic map, we refer to [3], [8] and [2], respectively.

§ 1. Complex analytic subspaces with locally stable parametrizations and their deformations. Let X and Y be complex manifolds, and S and T finite subsets of X and Y, respectively. A multi-germ $f:(X,S)\to(Y,T)$ of a holomorphic map at S is an equivalence class of holomorphic maps $g:U\to Y$ with $g(S)=T$, where U are open neighborhoods of S in X. Throughout this paper we shall interchangeably use a multi-germ of f and a representative g of f. A germ of a parametrized family of multi-germs of holomorphic maps is a multi-germ $F:(X\times\mathbb{C}^r, S\times 0)\to(Y\times\mathbb{C}^r, T\times 0)$ of a holomorphic map such that $F(X\times t)\subset Y\times t$ for any t in some open neighborhood of 0 in $\mathbb{C}^r$. An unfolding of a multi-germ $f:(X,S)\to(Y,T)$ of a holomorphic map is a germ of a parametrized family of multi-germs of holomorphic maps $F:(X\times\mathbb{C}^r, S\times 0)\to(Y\times\mathbb{C}^r, T\times 0)$ such that $F(x,0)=(f(x),0)$ for $x\in X$. We say that an unfolding $F:(X\times\mathbb{C}^r, S\times 0)\to(Y\times\mathbb{C}^r, T\times 0)$ of a multi-germ $f:(X,S)\to(Y,T)$ of a holomorphic map is *trivial* if there exist germs of t-levels ($t\in\mathbb{C}^r$) preserving analytic automorphisms $G:(X\times\mathbb{C}^r, S\times 0)\to(X\times\mathbb{C}^r, S\times 0)$ and $H:(Y\times\mathbb{C}^r, T\times 0)\to(Y\times\mathbb{C}^r, T\times 0)$ with $G_{|X\times 0}=id_X$, $H_{|Y\times 0}=id_Y$, such that $H\circ F\circ G^{-1}=f\times id_{\mathbb{C}^r}$. We say that a multi-germ