41. *On a Generalization of MacPherson's Chern Homology Class. II*

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§0. **Introduction.** For a non-singular variety X, a characteristic class of X is defined to be that of its tangent bundle TX and any characteristic class of X is expressed as a polynomial of individual Chern classes of X, in which sense Chern classes are fundamental characteristic classes for the case of non-singular varieties. As for the case of singular varieties, there is at the moment no general notion available of characteristic classes, mainly because one cannot define the tangent bundle unlike in the smooth case, although there are some notions of tangents, such as "tangent cone" [7] and "tangent star" [3]. For the Chern class and the Todd class, there are singular versions; namely, Deligne-Grothendieck-MacPherson's theory C_* (abbr. *DGM-theory*) of Chern class [4] and Baum-Fulton-MacPherson's theory Td_* (abbr. *BFM-theory*) of Todd class [1] (also see [5]). They are both formulated as unique natural transformations from certain group (covariant) functors to the homology group (covariant) functor such that they satisfy certain "smooth condition" (see below). Motivated by MacPherson's survey article [5] and the formulations of DGM-theory C_* and BFM-theory Td_*, the author [8] extended DGM-theory C_* of (total) Chern class to a "DGM-type" theory C_{t*} of the Chern polynomial, which includes DGM-theory C_* as a special case. In this note we give a characterization of "DGM-type" theories of characteristic classes under certain conditions.

§1. **DGM-theory of Chern class and pushforward stability.** Let $\mathcal{CV}$ be the category of compact complex algebraic varieties and $\mathcal{Ab}$ be the category of abelian groups. Let $\mathcal{F}: \mathcal{CV} \to \mathcal{Ab}$ be the correspondence such that for any $X \in \mathrm{Obj}(\mathcal{CV})$ $\mathcal{F}(X)$ is defined to be the abelian group of constructible functions on X. If we define the pushforward $f_* := \mathcal{F}(f)$ for $f: X \to Y$ by $f_*(1_W)(y) := \chi(f^{-1}(y) \cap W)$, then the correspondence $\mathcal{F}$ becomes a covariant functor with this "topologically defined" pushforward [4]. Let $H_*(\ ; Z)$ be the usual Z-homology group (covariant) functor. Then Deligne and Grothendieck conjectured and MacPherson proved the following, using Chern-Mather classes and his graph construction method:

Theorem 1.1 (*DGM-theory C_* of Chern class*, [4]). *There exists a unique natural transformation $C_*: \mathcal{F} \to H_*(\ ; Z)$ satisfying "smooth condition" that $C_*(1_X) = c(TX) \cap [X]$ for any smooth variety X, where 1_X is the*

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