40. A Note on the Mean Value of the Zeta and L-functions. VII

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1. Let $E_2(T)$ be the error-term in the asymptotic formula for the fourth power mean of the Riemann zeta-function, so that

$$\int_0^T \left|\zeta\left(\frac{1}{2}+it\right)\right|^4 dt = TP_4(\log T)+E_2(T) \tag{1}$$

with a certain polynomial P_4 of degree 4. As has been pointed out already in the preceding note [3] of this series, Corollary 2 in it gives an alternative proof of Zavorotnyi's claim [5]:

$$E_2(T)=O(T^{(2/3)+\varepsilon}) \tag{2}$$

for any fixed $\varepsilon>0$. In fact this is simply a resultant of combining the corollary with the spectral mean of Hecke series ([4]):

$$\sum_{\kappa_j\leqq x} \alpha_j H_j\left(\frac{1}{2}\right)^4 \ll x^{2+\varepsilon}. \tag{3}$$

Here $\{\lambda_j=\kappa_j^2+(1/4), \kappa_j>0\}$ is the discrete spectrum of the non-Euclidean Laplacian on $SL(2, Z)$, and $\alpha_j=|\rho_j(1)|^2(\cosh \pi\kappa_j)^{-1}$ with the first Fourier coefficient $\rho_j(1)$ of the Maass wave form corresponding to λ_j to which the Hecke series H_j is attached.

Though (2) is the hitherto best result on the upper bound for $E_2(T)$ it is generally believed that $T^{(1/2)+\varepsilon}$ may be the true order of it. The aim of the present note is to study this problem from the opposite direction. Namely we are going to show that under a hypothesis of the type of nonvanishing theorems for automorphic L-functions one may deduce an Ω-result on $E_2(T)$ from [Theorem, 3].

To formulate the hypothesis we denote by $\{\mu_h\}$, arranged in the increasing order, the mutually different members in the set $\{\kappa_j\}$. And we put

$$G_h=\sum_{\kappa_j=\mu_h} \alpha_j H_j\left(\frac{1}{2}\right)^3.$$

Now we set out

Hypothesis A. *Not all G_h vanish.*

Then we have

Theorem. *Under Hypothesis A $E_2(T)=\Omega(T^{1/2})$ holds.*

We note that Zavorotnyi's argument [5] does not seem to be able to yield our theorem. Also the theorem should be compared with the Ω-result on the mean square of $|\zeta((1/2)+it)|$ due to Good [1] (for the latest develop-

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