39. Askey-Wilson Polynomials and the Quantum Group $SU_q(2)$

By Masatoshi NOUMI*) and Katsuhisa MIMACHI**)

(Communicated by Shokichi IYANAGA, M. J. A., June 12, 1990)

The *Askey-Wilson polynomials* are a 4-parameter family of q-orthogonal polynomials expressed by the basic hypergeometric series ${}_4\varphi_3$. As special cases, it provides various types of q-Jacobi polynomials such as little, big and continuous q-Jacobi polynomials. In this note, we report that a (partially discrete) 4-parameter family of Askey-Wilson polynomials is realized as "doubly associated spherical functions" on the quantum group $SU_q(2)$.

In [2], Koornwinder realized a 2-parameter subfamily of Askey-Wilson polynomials as *zonal* spherical functions on $SU_q(2)$ in *an infinitesimal sense.* Generalizing his arguments to *non-zonal cases*, we obtain a 4-parameter family of Askey-Wilson polynomials that are connected to these polynomials as Jacobi polynomials are to Legendre polynomials in the $SU(2)$ case. From this interpretation, we also derive an addition formula for Koornwinder's 2-parameter extension of the continuous q-Legendre polynomials. Details will be given elsewhere.

1. Throughout this note, we fix a real number q with $0<q<1$. The algebra of functions $A(G)$ on the quantum group $G=SU_q(2)$ is the $\boldsymbol{C}$-algebra generated by x, u, v, y with fundamental relations

$$\begin{cases} qxu=ux,\ qxv=vx,\ quy=yu,\ qvy=yv, \\ uv=vu,\ xy-q^{-1}uv=yx-qvu=1, \end{cases} \tag{1.1}$$

and the *-structure determined by $x^*=y$ and $v^*=-qu$. The quantized universal enveloping algebra $U_q(su(2))$ is the $\boldsymbol{C}$-algebra generated by k, k^{-1}, e, f with relations

$$\begin{cases} kk^{-1}=k^{-1}k=1,\ kek^{-1}=qe,\ kfk^{-1}=q^{-1}f, \\ ef-fe=(k^2-k^{-2})/(q-q^{-1}), \end{cases} \tag{1.2}$$

and the *-structure with $k^*=k$ and $e^*=f$. As for the Hopf algebra structure, we take the coproduct determined by

$$\Delta(k)=k\otimes k,\quad \Delta(e)=k^{-1}\otimes e+e\otimes k,\quad \Delta(f)=k^{-1}\otimes f+f\otimes k.$$

The algebra of functions $A(G)$ has a natural structure of two-sided $U_q(su(2))$-module. For each $j\in(1/2)\boldsymbol{N}$, there exists a unique $2j+1$ dimensional irreducible representation of G of highest weight q^j with respect to $k\in U_q(su(2))$. By V_j we denote the corresponding right $A(G)$-comodule with coaction R: $V_j\to V_j\otimes A(G)$. We fix a $\boldsymbol{C}$-basis $(v^j_m)_{m\in I_j}$ for V_j, with $I_j=\{j, j-1, \cdots, -j\}$, such that the differential representation takes the form

*) Department of Mathematics, College of Arts and Sciences, University of Tokyo.
**) Department of Mathematics, Nagoya University.