26. On Algebroid Solutions of Some Algebraic Differential Equations in the Complex Plane

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1. Introduction. The purpose of this paper is to generalize the results obtained for binomial differential equations ([9]) to general algebraic differential equations. Let a_{jk} $(j=0, 1, \cdots, n;\ k=0, 1, \cdots, q_j)$ be entire functions without common zero for which $a_{0q_0}\cdot a_{nq_n}\neq 0$. Put

$$Q_j(z, w)=\sum_{k=0}^{q_j} a_{jk}w^k,\ (q_j=\deg Q_j)$$

and we consider the differential equation (=D.E.)

$$\sum_{j=1}^{n} Q_j(z, w)(w')^j=Q_0(z, w) \tag{1}$$

under the condition

$$q_n+n>q_j+j\ (j=1, 2, \cdots, n-1). \tag{2}$$

We suppose that the D.E. (1) is irreducible over the field of meromorphic functions in $|z|<\infty$ and that it admits at least one nonconstant ν-valued algebroid solution $w=w(z)$ in the complex plane. We say that the solution w is *admissible* if

$$T(r, f/a_{nq_n})=o(T(r, w))$$

for $r\to\infty$, possibly outside a set of finite linear measure, where $f=a_{jk}$ $(j=0, 1, \cdots, n;\ k=0, 1, \cdots, q_j)$. For example, when all a_{jk} are polynomials, a transcendental algebroid solution of the D.E. (1) is admissible.

In this paper we denote by E a subset of $[0, \infty)$ for which $m(E)<\infty$ and by K a positive constant. E or K does not always mean the same one when they will appear in the following. Further, the term "algebroid" (resp. "meromorphic") will mean algebroid (resp. meromorphic) in the complex plane. We use the standard notation of the Nevanlinna theory of meromorphic ([3]) or algebroid functions ([6], [10], [11]).

2. Lemmas. In this section, we shall give three lemmas for later use.

Lemma 1. *Let v be a transcendental algebroid function such that v and v' have at most a finite number of poles. Then, for some positive constants K_1 and K_2 it holds*

$$M(r, v)\leqq K_1+K_2rM(r, v')\quad (r\notin E),$$

where $M(r, v)=\max_{|z|=r}|v(z)|$ ([5]).

Lemma 2. *Let g be a transcendental entire function. Then,*

$$M(r, g')\leqq 2M(r, g)^2\quad (r\notin E) \qquad ([4]).$$

Lemma 3. *The absolute values of roots of the equation*