23. *On the Exponentially Asymptotic Stability of a Perturbed Nonlinear System*

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(Communicated by Kôsaku YOSIDA, M. J. A., March 13, 1989)

1. Introduction. Consider the following system of ordinary differential equations, (N), and its perturbed system, (P):

$$(N) \qquad \dot{x}=f(t, x),$$

$$(P) \qquad \dot{y}=f(t, y)+g(t, y),$$

where $f(t, x)$ is continuous, a Lipschitzian with respect to x and $f(t, 0)=0$. Moreover, $g(t, y)$ is continuous and $g(t, 0)=0$. On (N), we assume that the zero solution, $x=0$, has some properties on the stability.

Many authors have studied above systems under the conditions on $g(t, y)$ so that (P) preserves the stability of (N) (cf. Hahn [1], Yoshizawa [2], Strauss and Yorke [3], [4], etc.). In this paper, we give an attention to the exponentially asymptotic stability. A well-known result on this stability is as follows:

Theorem 1.1. *Suppose that the zero solution of (N) is exponentially asymptotically stable. Moreover, suppose that $\|g(t, y)\| \leqq u(t)\|y\|$ in some sets and $\int_0^\infty u(t)dt<+\infty$. Then the zero solution of (P) is exponentially asymptotically stable.*

Our purpose in this paper is to extend conditions on $u(t)$ to more general ones.

2. Definitions and lemmas. Let R^n be the n-dimensionl real Euclidean space and $\|\cdot\|$ denotes the norm on R^n. Let $B_h=\{x \in R^n: \|x\| \leq h\}$ for any $h>0$, and let $R^+=\{t \in R: t \geq 0\}$. $C[X; Y]$ denotes the set of all continuous functions from X to Y, where X and Y are topological spaces. We also write $C[X]$ instead of $C[X; Y]$. Let $\mathrm{Lip}(x, L, D)=\{f \in C[R^+ \times D]: \|f(t, x)-f(t, x')\| \leq L\|x-x'\|$ in $R^+ \times D\}$, where D is a domain in R^n, and $x(\cdot\,; t_0, x_0)$, $y(\cdot\,; t_0, y_0)$ denote any solutions of (N), (P) passing through (t_0, x_0), (t_0, y_0), respectively.

Definition 2.1. The zero solution of (N) is exponentially asymptotically stable ([Exp. A.S]) if there exist $h>0$, $K>0$ and $c>0$ such that $\|x(t; t_0, x_0)\| \leq K\|x_0\| \exp(-c(t-t_0))$ for all $(t_0, x_0) \in R^+ \times B_h$ and $t \geq t_0$.

If the zero solution of (N) is [Exp. A.S.], then we obtain the following lemmas.

Lemma 2.2. *Suppose that $f \in C[R^+ \times B_h; R^n] \cap \mathrm{Lip}(x, L, B_h)$ and the zero solution of (N) is* [Exp. A.S]. *Then there exist a Liapunov function*